Jacques ALLAIN

The FIFTH DIMENSION

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Notations and Terminology

Notations :

$\underline{\Delta}$	equality by definition
$\simeq$	approximately equals
$\equiv$	identical to
$\cdot$	scalar product of vectors
$\wedge$	vector product
$*$	convolution product
$\leftarrow$	substitution
$\rightarrow$	thus, hence, consequently
$\in$	belonging to
$\mid\ \mid$	absolute value, modulus or lacking a dimension (depending on context)
[ ]	dimension or unit (measurement theory)
e	energy
s	visual space
t	time
j	imaginary unit
f	frequency
v	speed
u	unit, unless there is an indication to the contrary
$\vec{u}$	unitary vector
$\vec{x}$	vector radius of the original space : $\vec{x} = x_i\ \vec{u}_i$; (i = 1, 2, …, N)
$\vec{\nabla}$	Nabla operator : $\vec{\nabla}\ \underline{\Delta}\ \frac{\partial}{\partial x_i}\vec{u}_i$; (i = 1, 2, …., N)
$\delta(\vec{x})$	DIRAC function at the origin of coordinates : $\delta(\vec{x}) = \delta(x_1)\ \delta(x_2)\ \ldots\ \delta(x_N)$

Written conventions :

- The norm of a vector $\vec{v}$ is written v.
- The modulus of a complex value (z) is noted as $|z|$.

Abbreviations :

(En) n^{th} stated
(Pn) n^{th} postulate

Terminology :

***Visual space* :** space accessible to viewing.
***Free space* :** the part of visual space that contains no force field.
***Universal space* :** the spatial model of the universe.
***Metric space* :** space of a single nature.
***Affine space* :** space of a multiple nature.
***Order of space* :** the number of its dimensions.

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INTRODUCTION

The revelation behind this essay goes back to the year 1976 when I prepared the end-of-year paper for my studies at the Technical University of Istanbul. The theme was "The study of the system of units", physical in general, electrical in particular. I took the opportunity to review the broad outlines of physics in order to ask myself the following question : why are the measurement dimensions (base quantities) different from the spatial dimensions ?

I discovered the answer to this question long before starting to expound on it, a task I began in the autumn of 1994. I covered the development of mechanics, starting with GALILEO and NEWTON, cosmology starting with COPERNICUS, via the great building blocks on which modern physics is based.

During ten years of uncertain, sporadic work, very remote from the world of science, I did my research, without making prior assumptions and in complete tranquillity, into the finality of principles and how they were matched by experiments. In summary, I established that the GALILEO - NEWTON basics are correct and that certain rather far-fetched theories, such as relativity and the BIG BANG, are fleeting.

This memorandum is based on three consecutive ideas :

- That the universe consists of three primary *natures* (elements) : energy, space and time.
- That these natures cover every possible theory : from that of measurement to that of spatial models.
- Consequently, the equations of physics lack one independent variable : that of energy.

Before tackling the content of these ideas, it is important to state that :

- "The fifth dimension" was the title assigned to this work from the beginning and it remains unchanged.

- This account is aimed especially at readers who are familiar with theoretical physics.
- The original version of the main text (the three chapters) has been retained, with the exception of a few formulae or brief passages from which it was essential to quote.
- The themes tackled are presented in a unique fashion and are considered to be superior on an academic level, hence the absence of a bibliography.
- The appendices include details of calculation, memoranda and a few illustrative examples.

The first chapter contains :

- the epistemology of this essay, the physical architecture of the universe and its geometrical model ;
- the new system of measurement (quantities and units) and a terminological revision of the constants ;
- a description of five-dimensional space.

The second chapter :

- deals in the classic manner (without any attempt at relativism) with the effect of acceleration ;
- condenses and criticises the theory of relativity ;
- offers a different interpretation of HUBBLE measurement and subsequently invalidates the BIG BANG theory ;
- proceeds to a dimensional analysis of MAXWELL's equations ;
- proposes an analogy between gravity and electromagnetism.

The third chapter develops a mathematical tool consisting of :

- the digitisation of continuous variables ;
- the derivation and integration of digital functions ;
- the resolution of linear differential equations.

With respect to the appendices :

- The first is a schematic of the new system of measurement.
- The second explains the DOPPLER and the HUBBLE Effects.
- The third is an addition to the "acceleration effects".
- The fourth is a reasonable summary of :
  - EINSTEIN’s relativity (special and general) ;
  - the mathematical concepts employed by relativity ;
  - LORENTZ transformations.
- The fifth relates the MAXWELL equations.
- The sixth contains analogical and digital FOURIER transforms.

Finally, this content consists of five statements, five postulates and four propositions.

I. THEORY

I.1 Preliminaries

I.1.1 The Universe

Foundations :

Definitions :

1) ***The Universe*** consists of everything that exists independently of human consciousness.
2) ***The natural order*** is the set of rules that governs the organisation of the Universe.
3) ***Absolute vacuum*** is the absence of any form of energy.

Comment :

A vacuum, in the usual sense, is a place that is devoid of matter and of a strong field. As we understand it, a vacuum is the region of space that has minimal intensity (to be determined) of energy.

Postulates :

(P1) The human faculty of abstraction is limited to combinations of tangible objects.

Corollary :

The elementary mathematical concepts are, in essence, physical.

(P2) The direction of abstraction is from the simple to the complex. Simplicity is natural, complexity needs to be justified.

Statements :

(E1) The subject of physics is the study of the observable and measurable structure of the universe.

(E2) Infinity is an indeterminate mathematical term ($a \pm \infty = \infty$, $a \times \infty = \infty$ for $a \neq 0$). It is consequently excluded from physics.

Postulates :

(P3) The universe is unique, continuous (without an absolute vacuum), limitless (boundless), finite and ordered. Its order is noumenal.

(P4) Man – body and mind – constitutes an infinitesimal part of the universe. He is consequently devoid of faculties of an absolute or comprehensive nature.

Comment :

Physics is the science of perception, both rational and empirical. No theoretician, regardless of the opportunity presented by his ideas, can impose anything (a rule, behaviour or property) upon Nature. Nature is totally independent of our world of thought. A credible physicist always seeks to understand the natural order in terms of logic.

Primary natures :

Postulate :

(P5) The universe is the combination of a finite, supposedly unknown, number of natures known as "primary", that are noumenal, incomparable, independent and inseparable. Each consists of an equally unknown number of independent but comparable components.

Currently, only three natures have been revealed :

- **Energy** consisting of a single component ;
- **Triple visual space** consisting of : the horizon, altitude and depth ;
- **The** single **time** component.

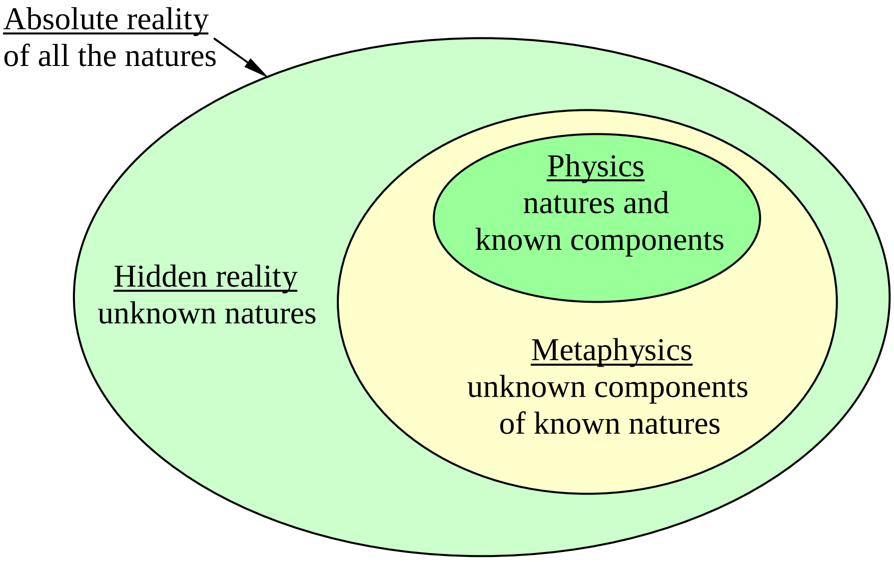
Areas of knowledge :

Four areas of knowledge have been defined :

- **Absolute reality** involving the whole universe, its natures and their constituents ;

- **Hidden reality** concerning unknown natures ;
- **Metaphysics** concerning the unknown components of known natures ;
- **Physics** consisting of the known components of known natures.

These areas are nested one within the other, in the following manner :



(Fig. I.1)

Comment :

Human knowledge belongs in the two inner rings (physics and metaphysics).

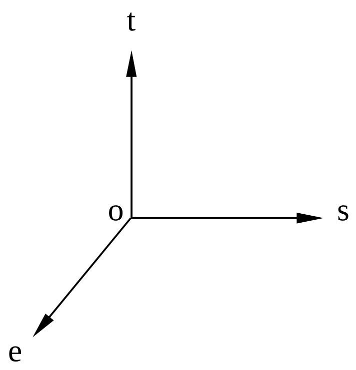
Thought :

The magnificent architecture of the universe, life and human intellect could not have occurred accidentally or as the result of a chaotic process. Another nature, currently unintelligible, manifests itself through this strange signature. This theory in no way supports theological theory or its metaphysical correlations.

I.1.2 Universal space

Structure :

By assigning a dimension to each of the primary natures, three-dimensional universal affined space (without the concept of distance) is evoked, illustrated by :



(Fig. I.2)

In this space, the measurements of the coordinates are not comparable and the geometrical structures are inalienable. By substituting s for its habitual components (x, y, z), one attains five-dimensional space (e; x, y, z; t), the subject of this account.

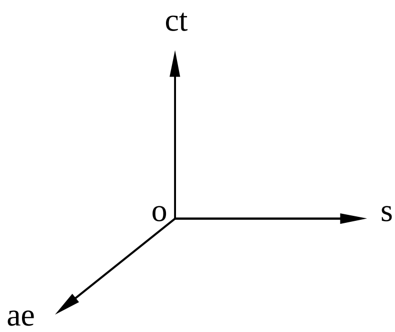
To convert affine space (e; s; t) into metric space (s, s, s) consisting of geometry and distance, the abstract unification of natures becomes indispensable. The appropriate nature for unification is visual space. Time and energy are subsequently expressed in terms of length.

The conversion formulae are the direct result of the ratios between the indivisible measurements of natures : Δt, Δs and Δe, known as a tribute as "PLANCK's natural units". The first two of these units are known literally whereas the third remains to be revealed (see § I.2.2).

The corresponding ratios and formulae are written :

$$\frac{\Delta s}{\Delta e} \;\underline{\Delta}\; a \quad ; \quad \frac{\Delta s}{\Delta t} = c \quad \Rightarrow \quad s = ae \quad ; \quad s = ct \qquad \text{(I.1)}$$

The triplet of (Fig. I.2) thus becomes :



(Fig. I.3)

It should be remembered that this (metric) space is only a conventional image of original space. In any case, it should be stated that the dimensions (or the variables of a function transposed in space) of any space must be absolutely independent of each other.

Geometry :

The first elements of so-called "classical" Euclidian geometry are the point and the straight line. A succession of points traces a line, a series of lines traces an area and so on. As to the shape, the curve is defined by variation in the straightness(1) and divided into two classes :

- a constant (circular) curve described by a point and a straight line ;
- a variable curve (regular or irregular) consisting of several points and straight lines.

Furthermore, it is accepted without need for a demonstration that every curve consists of a finite number of connected arcs.

Properties :

1) Curve geometry cannot accept straight lines. It is only tangible on the basis of an order of straight line geometry that is higher than its own.
2) Straight line geometry accepts any curve of a lesser order than its own.

In this context, and by virtue of the postulate (**P3**), universal space can only be curved and closed (limitless and finite). Its curve, according to the first of the above properties, is imperceptible. Furthermore, the principles of independence (**P5**) and simplicity (**P2**) imply the homogeneity of primary natures and the uniformity of their growth. In mathematical terms, the geodesics (see A.4.4) of natures are circular (perfectly symmetrical). This fundamental property of spatial geometry is more easily understood through the following example :

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(1) The corollary of the postulate (**P1**) suggests several definitions of the straight line. For example, the straight line is a luminous line crossing an empty region or the radial support of an electric field in an elementary charge placed in an empty environment, etc …

Let there be a closed surface (a two-dimensional space) containing a physical state that is initially freely distributed such as heat or an electrical charge. The only geometry capable of satisfying the requirement for homogeneity of this distribution is the constant curve.

Furthermore, the correspondence (unity and convertibility) between natures imposes the equality of their geodesic circles. This produces the sphericity of universal space as transcribed into the above statement.

Statement :

(E3) The geometrical properties of universal space are identical in every respect.

Comment :

1) Universal space is empty, any physical state or action being merely a geometric structure combining three projections relative to the primary natures, a *spatial* projection analogous to the "container", an *energetic* projection similar to the "content" and a *temporal* projection recalling the age.

2) On a human scale of observation and measurement, universal space is Euclidian.

I.2 Quantity and measurements

I.2.1 Base quantities

"Physical quantity" is the term applied to any object that is quantifiable in nature. These quantities are divided into two categories :

- ***base quantities*** that are irreducible and defined in themselves ;
- ***derivative quantities*** are composed of base quantities and defined by their initial formulae.

The constants produce a particular case : they are the ultimate values of ratios or experimental data.

Dimensions and modes :

Each fundamental quantity is assigned a capital letter known as the "dimension". The dimensional attribute of a derivative quantity indicates its physical composition. The constants resulting from the experiment are given weighted dimensions.

For reasons of consistency and subtlety, it is a good idea to introduce a new criterion for quantities. The "mode", denoted M^n, is the extensive ($n > 0$), intensive ($n < 0$) or neutral ($n = 0$) nature(1) of a quantity; n being the sum of the powers of the dimensional attribute (see A.1.2). Note that a quantitative quantity (with a non-selective dimension) is by definition extensive, the primary natures thereof constituting the example.

Statement :

(E4) The way in which the laws of physics are formulated must satisfy modal equality minus the constants.

(1) When this terminology is applied to thermodynamics, it changes its meaning.

Current quantities :

"Non-relativist" physics contains seven base quantities (see A.1.1) :

- three of mechanical origin : length (s), mass (m) and time (t), dimensioned [S], [M] and [T] respectively ;
- four of them originating from a particular branch[1] of physics : the electrical charge (q dimensioned as Q) for electricity, temperature (θ dim. Θ) for heat, etc ...

Quantities proposed : (see A.1.2)

The central idea of this account is the unification of primary natures and fundamental quantities, in other words, the merger of spatial dimensions and of measurement. This process leads to the harmonisation of the vital concepts of physics and consequently reduces the number of base quantities to three : encoded (e) and dimensioned [E] energy, length and time, and this applies to all of the branches together.

This idea emerged as follows :

Energy being the quantity shared by all areas of physics, it becomes a suitable substitute for the specific base quantity in each discipline. In order to use it in this way, the quantities must be expressed as a function of (e; s; t) in the form of equality, identity or equivalence. The quantities mentioned in this account are mass, electrical charge and temperature. Note in passing that temperature, like all other intensive quantities[2], cannot be fundamental.

The following stage consists in re-dimensioning all of the derivative quantities in terms of EST. For this purpose, the following Statement has been postulated.

Statement :

(E5) Two sizes having the same action are measured in the same way.

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(1) Each branch is distinguished by its own fundamental quantity.
(2) This type of size is derivative by definition.

Mass : m

Mass is defined by EINSTEIN's Theory of Relativity as :

$$E = mc^2 \Rightarrow [m] = ES^{-2}\, T^2 \qquad \text{(I.2)}$$

Electrical charge : q

Definition formula :

To quote NEWTON's law : $F_N = \dfrac{m_1\, m_2}{4\pi\, \gamma_o\, s^2}$ (I.3a)

and that of COULOMB : $F_C = \dfrac{q_1\, q_2}{4\pi\, \varepsilon_o\, s^2}$ (I.3b)

so let us consider two identical masses (m, m), separated by distance (s) and having two equal electrical charges (q, q). What is the ratio in which $\dfrac{m}{q}$, the forces F_N and F_C, cancel each other out ?

From the laws (I.3), the following can be extrapolated :

$$Q \equiv \sqrt{\frac{\varepsilon_o}{\gamma_o}}\; m \qquad \text{(I.4a)}$$

Since : $\dfrac{1}{4\pi\, \gamma_o} \simeq 6.67256 \times 10^{-11}$ [m³/kg s²] and

$\dfrac{1}{4\pi\, \varepsilon_o} \simeq 8.987 \times 10^{9}$ [m³ kg/C² s²], resulting in :

$$q \equiv 8.610425 \times 10^{-11}\ m \qquad \text{(I.4b)}$$

Dimension :

Dimensional equality and the perfect analogy (term to term) between the laws (I.3) implies : $[m] = [q]$ and $[\gamma_0] = [\varepsilon_0]$.

Corollary :

The formulae (I.2) and (I.4b) link electrical charge to energy through :

$$E = \zeta q \quad ; \quad [\zeta] = S^2\, T^{-2} \quad ; \quad \zeta \simeq 1.0452446 \times 10^{27}\ [j/C] \qquad \text{(I.5)}$$

making it possible to deduce therefrom that :

- the indivisible(1) electrical charge in nature is :

$$\Delta q \simeq 1.321 \times 10^{-50} \text{ [C] ;}$$

- the electrostatic energy of the electron amounts to :

$$1.6746796 \times 10^{8} \text{ [j] .}$$

It is instructive to note that the electrical charge is the most concentrated form of energy known hitherto.

Temperature : θ

The analogy between PLANCK's formula :

$$E_P = h\, f = \hbar\omega \qquad \text{(I.6a)}$$

and that of BOLTZMANN : $E_B = k\,\theta$ (I.6b)

induces : $[k] = [h] = ET$ and $[\theta] = [f] = T^{-1}$. This last equality also results from WIEN's law : $f_m = 1.034552 \times 10^{11}\,\theta$.

I.2.2 Units of measurement

The literature covers two series of units :

- a conventional series adapted for human usage ;
- another so-called "natural" series consisting of PLANCK's measurements.

Conventional units :

The majority of authors use the International System of Units that covers the seven fundamental quantities quoted in (A.1.1). Our model (see A.1.2), which refers to new fundamental quantities, comprises three units : [jms] for joules, metres and seconds. Consequently, the unitary equivalents of the old fundamental quantities are printed as :

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1) This charge comes from the unit of heat ($\mathbb{k}$), see the next paragraph. Likewise, the elementary mass can be deducted : $\Delta m \simeq 1.5341 \times 10^{-40}$ [kg] .

$1\ \text{kg} \equiv 9 \times 10^{16}$ [js²/m²] for mass, $1\ \text{C} \equiv 1.04256 \times 10^{27}$ [js²/m²] for electrical charge and $1\ °\text{K} \equiv 2.08365 \times 10^{10}$ [1/s] for temperature(1).

Proposition :

Starting from an intrinsically defined "second", it is advisable to have a "metre" that corresponds very precisely to $c = 3 \times 10^{8}$ [m/s]. This having been done, it can continue to be stated with certainty that :

$$\varepsilon_0 = \frac{10^{-9}}{36\pi}\ [\text{F/m}] \text{ for a value of } \mu_0 \triangleq 4\pi \times 10^{-7}\ [\text{H/m}] \text{ already agreed.}$$

This measurement appears to be simultaneously convenient, accurate and consistent.

Natural units

These are the measurements Δe, Δs and Δt defined as follows :

$$\Delta t \triangleq \sqrt{\frac{\hbar G}{c^5}} \simeq 5.39 \times 10^{-44}\ [\text{s}] \ ; \ \Delta s \triangleq c\Delta t \simeq 1.616 \times 10^{-35}\ [\text{m}] \quad (\text{I.7a})$$

As for Δe, which is currently unknown, it is perceived as follows :

A process is known as "periodical" if it is repeated at regular intervals in the direction of its development. Such a process must consist of at least two successive periods and is defined over a technically whole period known as the "principal period". The frequency (f) thus designates the number of periods contained in the unit of measurement (a second, in the case of time).

In this context, PLANCK's formula (I.6a) would appear to show the following :

1) The energy of a period corresponds to :

$$f = 1\ [\text{Hz}] \ \Rightarrow\ E(1) = |\,h\,|\ [\text{j}]$$

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(1) The round figures of $1\ \text{C} \simeq 10^{27}$ [js²/m²] and $1\ °\text{K} \simeq 2 \times 10^{10}$ [1/s] are tolerable.

2) This energy is independent of the frequency and is indivisible and thus unitary :

$$\Delta e \;\simeq\; 6.626 \times 10^{-34}\ [j] \qquad \text{(I.7b)}$$

3) The "quantum" of an electromagnetic wave (or any wave that propagates itself at the speed of light) constitutes the sum of the energy of f periods; f obviously represents a whole number.

In order to release f from the constraint of being a whole number and rendering PLANCK's formula (I.6a) more consistent, the following equation is proposed :

$$\mathscr{I}_P \;=\; \mathbb{h}\, f\ [j/s] \quad ; \quad \mathbb{h}\ \underline{\Delta}\ \Delta e \;\; ; \qquad \text{(I.8a)}$$

$\mathscr{I}_P$ being PLANCK's radiation intensity.

By analogy, BOLTZMANN's radiation intensity is :

$$\mathscr{I}_B \;=\; \mathbb{k}\, \theta\ [j/s] \quad ; \quad \mathbb{k}\ \underline{\Delta}\ |\, k \,|\ [j] \;\; \text{(unit of heat)} \qquad \text{(I.8b)}$$

This unit characterizes the least action defined by :

$$\mathbb{k}\, \Delta t \;\simeq\; 7.442 \times 10^{-67}\ [js]$$

Corollary :

An electromagnetic (or gravitational) wave is not propagated in space (e; s; t) except as a multiple of its period.

Comment :

1) Discrete measurement (in PLANCK units) of spatial lines does not contradict their continuity. These units are like sliding segments on the geodesics.
2) Quanta and photons, as perceived by PLANCK and EINSTEIN, do not exist. Any radiation, whether intense or otherwise, follows in the wake of Δe.

I.2.3 Constants

Constants are only superficially expressed in the literature. Out of a concern for accuracy, here are the redefinitions :

number, any constant having no dimension (value independent of the scale of measurement) ;

absolute constant, any constant without a mode ;

fundamental constant, any absolute, irreducible and experimental constant ;

measurement, any other constant ;

measurement limit, any extreme measurement in nature ;

fundamental measurement, any irreducible and experimental measurement.

In accordance with this terminology, current physics only counts three fundamental constants corresponding to a vacuum :

γ_o gravitational property (NEWTON's constant) ;

ε_o electrical property (absolute permittivity) ;

μ_o magnetic property (absolute permeability).

The fundamental measurements consist of :

$\mathbb{h}$ unit of energy (PLANCK's modified measure) ;

H_o minimum frequency (HUBBLE's measurement).

Comment :

Certain derived constants, such as c and k, are poorly expressed in the documentation. It is useful to recall that :

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \quad ; \quad k = \frac{R}{N_A}$$

with : $R \simeq 8.3144$ [j/(mol.°K)], as the molar constant of perfect gases ;

$N_A \simeq 6.02204 \times 10^{23}$ [1/mol], the AVOGADRO number.

***New constants* :**

The ratios $a = \frac{\Delta s}{\Delta e} \simeq 2.4386333 \times 10^{-2}$ [m/j] and $b \triangleq \frac{\Delta e}{\Delta t} \simeq 1.22934619666 \times 10^{10}$ [j/s] represent two maxima of radiation : the first is the inverse of the longitudinal density of radiation whereas the second indicates its intensity.

I.3 Five-dimensional Space

Original version :

Description :

Space (ae, s, ct) is five-dimensional and it is real, continuous, universally spherical and locally Euclidian. Its metric is expressed on the universal scale :

$$ds^2 = g_{pq}(x^1, x^2, x^3, x^4, x^5)\, dx^p\, dx^q \quad ; \quad x^1 = ae \quad ; \quad x^5 = ct \qquad \text{(I.9a)}$$

and $ds^2 = a^2de^2 + dx^2 + dy^2 + dz^2 + c^2dt^2$ on the local scale. (I.9b)

Geometrical figures :

Each physical object can be identified by a unique geometrical body in space (ae, s, ct). Most authors confuse corporeal geometry with spatial geometry. They are in the habit of using the geometry of the body they are considering as "measurement space" to describe "configuration space"(1) or merely space(2) on its own.

Functions :

Any function originating from physics is of the $\Xi(e; s; t)$ type. Its value at a given point $(e_o; s_o; t_o)$ in space is unique(3).

Discrete version :

In terms of infinitesimal measurement (on the PLANCK scale), spatial lines are shown to be discrete.

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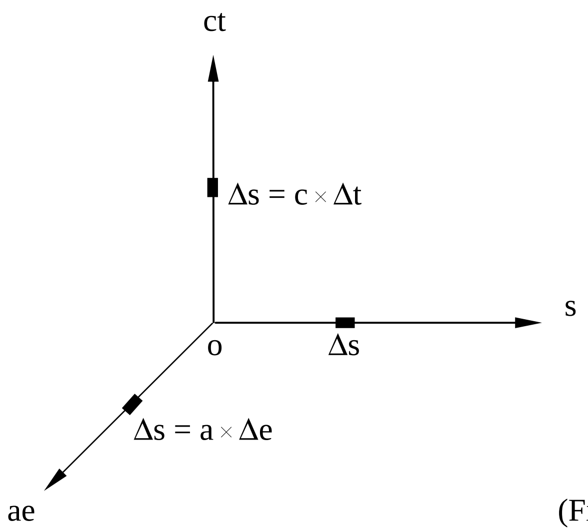
(1) Let there be a dynamic system with n degrees of freedom or $q^1, q^2, \ldots, q^n$, all of these parameters describing the way such a system develops. By assigning a coordinate to each of these parameters, a configuration space of n dimensions is created.

(2) This is the case with general relativity that transposes the geometry of gravitational action into spatial geometry.

(3) This is not the case for a function of (s; t).



(Fig. I.4)

taking the elementary steps :

$\Delta s \simeq 1.62 \times 10^{-35}$ [m] ; $\Delta t \simeq 5.39 \times 10^{-44}$ [s] ; $\Delta e \simeq 6.63 \times 10^{-34}$ [j]

and the constants : $c = 3 \times 10^{8}$ [m/s] ; $a \simeq 2.44 \times 10^{-2}$ [m/j]

It should be borne in mind that the indivisible units of derivative extensive quantities are deduced directly from their definition formulae.

From another point of view, the digitisation of spatial coordinates on an infinitesimal scale is only palpable in the tenuous world (at very low energy). Beyond this world, one resorts to the sampling technique (see III.1.1).

Differential equations :

Equations in physics are generally differential, local (functions of spatial coordinates), non-linear and only resolvable in very special cases. The introduction of energy as an independent variable in these equations renders them linear. Consequently, the digitisation expanded upon in chapter (III), combined with FOURIER’s analysis (A.6), leads to the solving of such equations as a general principle.

II. SUPPLEMENTS

II.1 Movement

Movement is quite rightly described as the motion (change in spatial position) of one body in relation to another. The name "scene" is given to the field of observation and the name "referential 0" to the supposedly motionless body in the background of the scene.

Two types of movement are detectable :

- A so-called "free or independent" movement (acceleration or clock movement) of an origin external to the scene. This type of movement contributes no information to the study.
- A movement, known as "dependent", originating within the scene, providing information.

Movement is characterised by direction and speed (v) that varies between immobility and the speed of light (c). This variation is often expressed by the ratio $(\frac{v}{c})$.

II.1.1 Light

Light is an electromagnetic wave consisting of five parameters : amplitude, phase, frequency, speed and direction of propagation. The last three parameters are of prime importance :

- *Frequency* : this very revealing shift is used in several areas of physics ;
- *speed* : an electromagnetic characteristic of a vacuum and an absolute constant, it classifies other speeds ;
- *trajectory* : according to EINSTEIN, describes spatial geodesics.

Furthermore, experiments show that :

1) The speed of light is isotropic, limited and non-additive :

$$c \pm v = c \quad ; \quad v \neq 0$$

2) Light interacts with other forms of energy :

- *movement* : light loses or gains energy (its frequency increases or decreases) when it leaves or encounters motion (the DOPPLER Effect (see A.2.1)) ;
- *field* (electromagnetic or gravitational) :
  - light loses or gains energy when it leaves or encounters a celestial body, the EINSTEIN Effect ;
  - it deviates when close to an astral body (EINSTEIN's proposition), the HUBBLE Shift (see A.2.2) is the result of this deviation.

II.1.2 Effects of acceleration

The experimental data show that acceleration (or any alteration to the state of energy in general) of a body changes some of its physical properties, such as its mass and clock rate(1). This change is explained by :

$$\text{State}_2 = \left[1-\left(\frac{v_f - v_i}{c}\right)^2\right]^{\pm\frac{1}{2}} \times \text{State}_1 \qquad \text{(II.1a)}$$

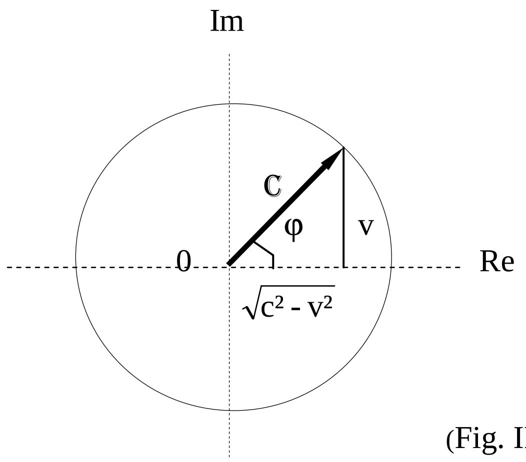
where v_i and v_f respectively represent the initial and final acceleration speeds. If any of these speeds is nil, the ratio (II.1a) becomes :

$$\text{State}_2 = \left(1-\frac{v^2}{c^2}\right)^{\pm\frac{1}{2}} \times \text{State}_1 \qquad \text{(II.1b)}$$

Formulation :

In a complex plan, let us combine the speed of a mobile object with that of light :

(1) One asks oneself whether other properties change in parallel, the electrical charge or the temperature for example.

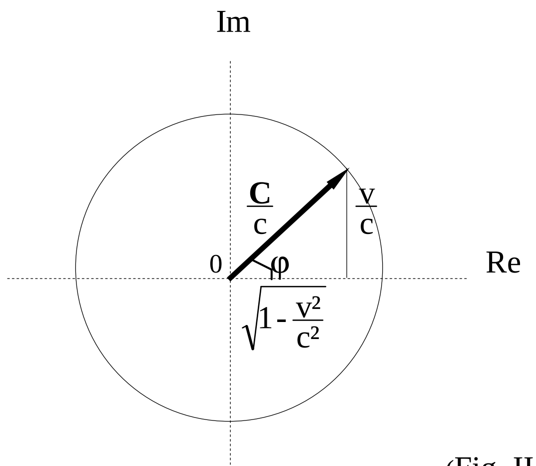


(Fig. II.1)

where : $\mathbb{C} \;\underline{\Delta}\; c \exp(j\varphi)$ is a phasor rotating in the positive direction of the acceleration ;

$$\varphi \;\underline{\Delta}\; \left(\arcsin \frac{v}{c}\right) \in \left[0, \frac{\pi}{2}\right[.$$

Then let us convert this "circle of speed" into a unitary circle of speed :



(Fig. II.2)

with : $\left|\frac{\mathbf{C}}{c}\right| = 1$

It will soon be realised that the ratio $\frac{v}{c} = \sin\varphi$ evokes the DOPPLER effect and that :

$$\frac{\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} = \operatorname{tg}\varphi \qquad \text{(II.2)}$$

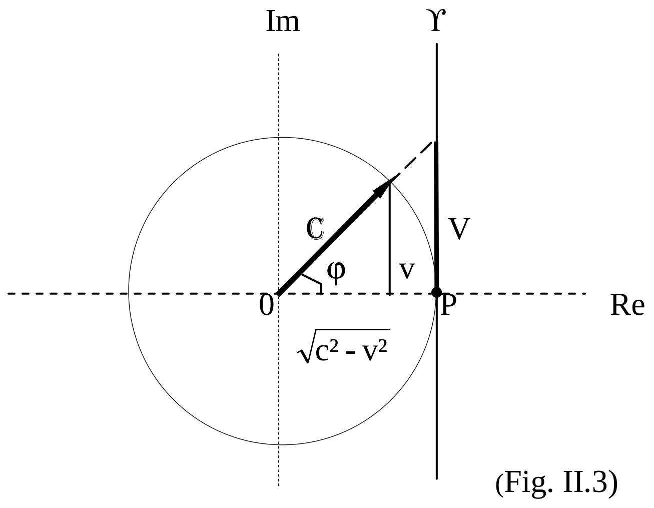
produces the acceleration effect that can be calculated in various ways.

It should be emphasised that this effect is cumulative and that the substitution, in the basic formulae, of $\frac{v}{c}$ by $\frac{\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$ or quite simply :

$$v \leftarrow \frac{v}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \text{(II.3)}$$

which is perfectly acceptable without resorting to space-time relativity.

From the analytic point, the radial projection of v on the tangent to the circle at the resting point (P) produces :



(Fig. II.3)

"Tangential speed" (or effective speed) is the quantity V defined by :

$$V \triangleq \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{V}{c} = \mathrm{tg}\,\varphi$$

playing the same role as v in the traditional formulae of movement.

II.1.3 Kinetic Applications

Kinetics is the study of movement solely in relation to (s; t). The kinetic effect of acceleration is manifested in two typical cases : the composition of dependent speeds(1) and the slowing of clocks.

***Composition of speeds*(2):**

Let us consider a cascade (i = 1, 2, ..., n) of hierarchical movements (each referring to the previous one), in a straight line and unidirectional. "Actual speed", denoted as v_i , is the name given to the speed of the i^{st} body in relation to $(i-1)^{st}$; v_1 thus designates the speed of the first body in relation to the scene as a reference point 0. This having been done, let us ask the following question :

What is the "residual" speed between two cascading bodies of any kind, especially the last in relation to the scene ?

The answer has already been provided in the ratio (II.2). If v is the speed of the n^{th} body in relation to the scene :

$$\mathrm{tg}\,\varphi = \sum_i \mathrm{tg}\,\varphi_i \quad ; \quad \varphi_i = \arcsin \frac{v_i}{c} \Rightarrow v = c \sin\varphi$$

---

(1) An example illustrating independent velocities is provided in (A.3.1).

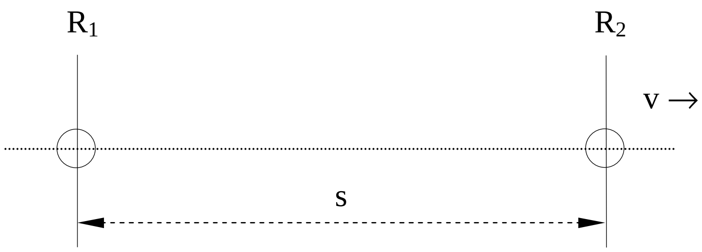
(2) Scalar composition of uniform velocities, easily extendable to vectorial cases.

This formula upsets the famous "law of composition"[(1)] of two velocities that is heavily promoted in the literature.

Slowing of clocks :

Experiments show that acceleration or exposure to a strong field (gravitational or magnetic) slows the movement of clocks. Take two references of explorers (R_1 and R_2) equipped with two identical clocks and placed on a straight and graduated line of action. At the outset, the two explorers coincide in space (s; t) and agree to work together to conduct the following experiment :

From the moment ($t_1 = t_2 = 0$), R_2 leaves R_1 at a constant (v) raised speed[(2)], after an agreed period of time has elapsed (t), the distance (s) that separates him from R_1 is represented as follows :



(Fig. II.4)

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(1) According to this "law", the ratio $\frac{v}{c}$ is comparable (or equal to) th φ with :

$$\varphi = \varphi_1 + \varphi_2 \;\Rightarrow\; \text{th}\,(\varphi_1 + \varphi_2) = \frac{\text{th}\,\varphi_1 + \text{th}\,\varphi_2}{1 + \text{th}\,\varphi_1\,\text{th}\,\varphi_2} \;\Rightarrow\; v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

For our part, we accept that : $\frac{v}{c} = \sin\varphi$ and $\text{tg}\,\varphi = \text{tg}\,\varphi_1 + \text{tg}\,\varphi_2$.

(2) Virtually instantaneous acceleration is assumed.

The immediate application of the first e law of kinetics ($s = vt$) is given for R_1 and R_2 : $s_1 = vt_1$ and $s_2 = vt_2$ respectively, but the pick-up of s by R_2 invalidates the calculation and renders suspect one of the three parameters used : s, v or t. Since s is invariable and v is the initial data, the error can only originate from the measurement of time : $t_1 = t$; $t_2 \neq t$.

In order to make the calculation match the experiment, R_2 resorts to a replacement (II.3) :

$$s_2 = \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} \times t_2 \underline{\Delta} V t_2$$

Enabling him to match his time scale to that of R_1 :

$$\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} \times t_2 = vt_1 \Rightarrow t_1 \equiv \frac{t_2}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{(II.4)}$$

This is the equivalence of the measurement of time between two reference points, one moving in a manner dependent on the other. The same result will be reproduced if the experiment is reproduced over an agreed distance (s) and a comparable pick-up of t_1 and t_2 .

Time scales :

The result of the previous paragraph is that each reference point (whether in motion or not) has its own time scale (the clock and its mechanism) and the dependent movements cause dependence on the scales.

Let us consider a sequence of dependent cosmic movements starting from the reference point 0 (presumed to be an initial resting state) and moving towards a reference point in motion at a speed close to c. It can be assumed that the clock 0 moves at maximum speed even though it has virtually stopped. If this is the case, the formula (II.4) can be extended to any pair of consecutive movements :

$$t_{n+1} \equiv t_n \sqrt{1 - \left(\frac{v_{n+1}}{c}\right)^2} \quad ; \qquad \text{(II.5)}$$

v_{n+1} being the speed of R_{n+1} in relation to R_n .

In fact, it should be remembered that this sequence of scales terminates in the minima (Δt) and maxima $\left(\frac{1}{H_o}\right)$ of our own time scale.

Units of time :

Take two reference points (R_1 and R_2) using the same technique (clock and definition of a unit) for measuring time. The animation of R_2 in relation to R_1 generates a difference of scales represented by the ratio (II.4) :

$$u_1 \equiv u_2 \sqrt{1 - \frac{v^2}{c^2}} \qquad \text{(II.6)}$$

where u_1 and u_2 are the units of measurement in relation to R_1 and R_2.. The result is that one second on the scale of R_1 corresponds to $\sqrt{1 - \frac{v^2}{c^2}}$ seconds on the scale of R_2 . Note that the difference in scale changes the values of those constants having a time dimension, the speed of light for example.

Shift of scales :

The spectral shift is caused by the difference in scales between the two units of electromagnetic transmission, one being mobile in relation to the other (Fig. II.4). The period $T_1 = \frac{1}{f_1}$ emitted by R_1 reaches R_2 in the form of $T_2 = \frac{1}{f_2}$. This shift is caused by virtue of (II.4) at :

$$T_2 = T_1 \sqrt{1 - \frac{v^2}{c^2}} \;\Rightarrow\; f_2 \equiv \frac{f_1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{(II.7)}$$

It should be added that this ratio remains valid regardless of the direction of transmission between R_1 and R_2, the type of movement (dependent or free if the ratio between the scales is known) and the direction of movement (approaching or receding).

Similarly to the DOPPLER shift, this produces :

$$\Delta f \;\underline{\Delta}\; \left| f_e - f_r \right| = f_2 - f_1 = f_1 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) = f_2 \left[1 - \sqrt{1 - \frac{v^2}{c^2}} \right]$$

Besides, it is possible to combine (couple) shifts of the same type (spectral for example).

Coupling the shifts : *DOPPLER and scales*

From the diagram (Fig. II.4), four modes of transmission can be extrapolated relative to the direction of movement and the permutation of the emission/reception between R_1 and R_2 :

Case of receding movement :

1) R_1 transmitter, R_2 receiver ; $f_1 \Leftarrow f_e$; $f_2 \Leftarrow f_r$

   The frequency f_e reaches R_2 having undergone two effects :

   * the DOPPLER Effect which converts it into : $f_e \left(1 - \frac{v}{c}\right)$ and then
   * the scalar effect (II.7) which brings it to :

$$f_r = f_e \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$$

2) R_2 transmitter, R_1 receiver ; $f_2 \Leftarrow f_e$; $f_1 \Leftarrow f_r$

The frequency f_e subject to the shift in scales leaves R_2 with the DOPPLER effect to become :

$$f_r = f_e \sqrt{1 - \frac{v^2}{c^2}} \times \left(1 - \frac{v}{c}\right)$$

Case of approaching movement :

Through a consideration comparable to the previous one, it is found that :

1) for R_1 transmitter, R_2 receiver ; $f_r = f_e \dfrac{1 + \frac{v}{c}}{1 - \frac{v}{c}}$

2) for R_2 transmitter, R_1 receiver ; $f_r = f_e \sqrt{1 - \frac{v^2}{c^2}} \times \left(1 + \frac{v}{c}\right)$

Critical speed :

The results of coupling show that :

- in a case of receding movement, f_r is always less than f_e ;
- in a case of approaching movement and for the R_1 transmitter, f_r is always greater than f_e .

The only case in which the two shifts (DOPPLER and scalar) may cancel each other out ($f_r = f_e$) is that of "approaching the R_2 transmitter". Consequently :

$$\frac{v}{c} \simeq .7709169971 \;\Rightarrow$$

$$v \simeq 2.312751 \times 10^8 \text{ [m/s]} \;\underline{\Delta}\; v_c \text{ (critical speed).}$$

This value, applied to the schematic (Fig. II.2), represents :

$$\varphi \simeq .88 \text{ [rad]} \simeq 50.43631 \text{ [°]}.$$

II.1.4 Dynamic Applications

Dynamics is the study of movement on the basis of (e; s; t). NEWTON was the first to formulate the first law of dynamics :

$$F = m_o \frac{dv}{dt} \text{ ; } m_o \text{ being the static mass of the mobile object.}$$

The validity of this law is restricted to low acceleration. In the general case, the replacement (II.3) is required :

$$F = m_o \frac{d}{dt}\left[\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right] = m_o \left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}} \times \frac{dv}{dt}$$

The first implications of this law affect impulse and kinetic energy (see the calculation in A.3.2).

Impulse (quantity of movement) :

Definition formula : $J \underline{\Delta} \int_0^t F\,dt$

Ordinary expression : $J = m_o v$

Generalised expression : $J = m_o \frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$

It is plausible to directly substitute m_o with $\frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$ in the ordinary expression in order to obtain the generalised expression. The latter includes the increase in mass caused by acceleration :

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{(dynamic mass).}$$

***Kinetic energy* :**

Definition formula : $E_c \ \underline{\Delta} \int_0^s F\,ds$

Ordinary expression : $E_c = \frac{1}{2} m_o v^2$

Generalised expression : $$E_c = m_o c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \qquad \text{II.8)}$$

The last equation contains three forms of energy :

- Static (motionless) energy known as potential energy :

$$E_o = m_o c^2$$

This is the emblematic relationship of restricted relativity.

- Dynamic or total energy : $E_t = m_o \frac{c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$
- Kinetic energy : $E_c = E_t - E_o$

The serial development of the formula (II.8) obtains, in the case of low speeds, the ordinary expression of E_c .

II.2 Relativity

Before the discovery of electricity, NEWTON's so-called classical mechanics dominated physics and astronomy. This discipline, based on the movement of material bodies, associated energy with mass via force or impulse. The advent of electromagnetism upset the world of physics at the time, by introducing a new form of non-material energy that propagated itself at the speed of light. This significant breakthrough in experimentation stimulated the theoreticians in their search for a new scheme of physics incorporating electromagnetism and, at a later stage, quantum energy.

For more than a century, the researchers multiplied their efforts without their achieving any real measure of success. In the first two decades of the twentieth century, Albert EINSTEIN maintained a thesis that became a veritable classic, one that provided the answer to several insoluble issues of the time. He imagined the space-time-matter interaction and built his theory (relativity) in two phases : "special" and "general". The first interprets the effects of acceleration in which the ratio $\frac{v}{c}$ becomes significant while the second creates a quadri-dimensional (quadri-variable to be precise) equation between the surfaces of gravitational equi-forces.

II.2.1 Special Relativity (see also A.4.2)

Reminder :

Mechanics, both rational and empirical[1], were introduced in 1638 by GALILEO and supplemented in 1686 by NEWTON. GALILEO was responsible for the relativity of movement *"a movement has no direction unless it is related to a defined system of reference"* and the first principle of invariance concerning the laws of movement.

(1) rational (of thought) and empirical (of knowledge).

It is to him that the first group of conversions of coordinates between two reference points is owed, one being a translational rectilinear movement that is uniform in relation to the other.

The GALILEO Group remained unshakeable until 1872, the year in which MAXWELL (see A.5) established his equations concerning electromagnetism. These invalidated GALILEO’s transformations (due to their variance) and encouraged LORENTZ to pursue research into a new group valid for the whole of physics, and for electromagnetism in particular.

The experiments conducted by MICHELSON-MORLEY in 1887 added a new embarrassment to the old laws of mechanics when he publicly and incontrovertibly demonstrated that the speed of light is both limited and non-additive. This overturned the traditional law of the vectorial composition of velocities.

In order to revive physics, LORENTZ simply renounced the fundamental concepts of Newtonian mechanics and formulated the second "principle" of invariance concerning the speed of light. He believed that space-time interacts with the movement of bodies and changes its properties in relation to the ratio $\frac{v}{c}$. This change was the basis for his own group of transformations established in 1903 and validated by the invariance of MAXWELL’s equations.

LORENTZ’s space-time model attracted the interest of MINKOWSKI who suggested the Euclidian quadri-dimensional continuum (vacuum) and complex[(1)].

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(1) The time dimension was considered to be imaginary so that the metric would remain invariable in the LORENTZ group. It is pitiful to see the relativists devalue the GALILEO group simply because GALILEO did not satisfy the invariance of the MAXWELL equations while they support the LORENTZ group to the detriment of the metric which needs to remain real.

Discussion :

Principles :

The first principle of special relativity is of an aesthetic nature since the laws of physics are, by their nature, independent of any form of expression.

As for the second "principle", the speed of light is an electromagnetic measurement in a vacuum. It is invariable in essence and unworthy of formulation in a principle.

Hypothesis : (The LORENTZ Contraction)

The LORENTZ Contraction is merely an arrangement of calculus. Let us consider a space shuttle equipped with an optical instrument for measuring length. According to the two principles of relativity, the length of the shuttle is measured when it is at rest and at very high speed $\ell = ct$ and $\ell' = ct'$ respectively. The $\ell' < \ell$ inequality is interpreted in the first degree, as a contraction of the shuttle. This hypothetical contraction must be independent of the direction of movement.

Implications :

1) Time does not dilate with acceleration, these are clocks that are slowing down.
2) The LORENTZ group, obtained as an example in (A.4.5), does not refer to any event and does not use any law as a criterion of validity. On the contrary, a system of transformation of coordinates must be transparent to all the laws including the fundamental metric (I.9).
3) The geodesics of space-time are independent of any movement.

Formalism : (MINKOWSKI metric)

The MINKOWSKI metric is a mathematical expression devoid of physical meaning[(1)]. It should be specified that MINKOWSKI space is not four-dimensional but three-dimensional with a time parameter.

II.2.2 General relativity (see also A.4.3)

Reminder[(2)] :

Albert EINSTEIN, who was aware of the inadequacy[(3)] of the Newtonian theory of gravity and enthusiastic about LORENTZ's "relativist" ideas, embarked on the search for a spectacular but more general alternative[(4)]. He was inspired by two principles, an ancient one dating back to the Greeks and a new one. The first, borrowed from DESCARTES via MACH, saw space as a support for matter, the second (1907) claimed to create the equivalence between Galilean inertia and the gravitational force.

EINSTEIN, in his Special Theory of Relativity, generalised the principle of invariance : *"all of the systems of GAUSS coordinates are equivalent for the formulation of the laws of nature"* and gave it a mathematical formulation (the covariance principle) : *"the laws of nature need to be co-variant in relation to any continuous transformations of coordinates"*. He predicted some of the effects of gravity and reached the pinnacle of his achievements by demonstrating that NEWTON's theory was just a limitation[(5)] of his own.

---

(1) since the formulation of nature must be real.
(2) We will concentrate exclusively on the original version of relativity without taking account of extrapolations added subsequently.
(3) such as the inexplicable precession of the perihelion of the solar planets which is particularly notable in the case of Mercury.
(4) valid at any point in space-time, regardless of the distribution of matter and the movement thereof.
(5) The two theories agree at points distant from the source of the field.

The effects of gravity :

EINSTEIN stipulated the distortion of light passing close to a heavenly body : "*a ray of light in a gravitational field must become curved similarly to the curvature of the trajectory of a body thrown through a gravitational field*". He realised there would be a loss of energy (the spectral shift to the red) of radiation in a gravitational field : "*an electromagnetic wave emitted by a massive body at a given frequency will propagate in space at a frequency that is shifted downward*". He claimed that clocks would slow down if exposed to a strong field : "*clocks will run more slowly if they are located in the vicinity of a heavy mass*" and specified : "*the frequency of an atom on the surface of a celestial body is slightly smaller than the frequency of an atom of the same element that is found in free space or on the surface of a smaller celestial body*".

Success :

Relativity correctly explains the precession of the perihelion of Mercury and has passed all the tests to which it has been put.

Discussion :

Principles :

The MACH Principle contradicts the postulate **(P5)**. Despite the perfectly acceptable criticisms of the relativists, the Newtonian vision of the universe remains inevitable.

Covariance is cannot be defined as a principle, it is the tensorial property of invariance.

The "principle of equivalence" dedicated to the hypothesis of the curve of space is an argument of a proposition, valid uniquely as a model. The heavenly bodies are subject to a balance of forces and are not free nor are their movements inert.

Hypothesis : **(**curve of space)

Mass does not cause space to curve, since interaction between different natures is prohibited. As for the distortion of light, considered to be proof of the theory, this is the result of interference between the gravitational, electrical and magnetic fields. It should always be remembered that the curve of space, if it exists, would never be perceptible to man.

Formalism : (field equation)

The equation $$\frac{1}{2} g_{\mu\nu} R - R_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad \text{(II.9)}$$

is insoluble in this form since the coordinates[(1)] of its left and right members are unknown. EINSTEIN overcame this difficulty by resorting to the preliminary data (initial conditions) leading to the (special) solutions approached. This type of manipulation attacks the generality of the theory.

Moreover, the dimension $[S^{-2}]$ of the above equation implies a two-dimensional curve of nature (s). This is a surface that is manifestly closed, replacing the vectors of gravitation equi-forces at its extremities. This surface, whose topology depends upon the quantity of enclosed matter and its movement, confirms the laws of the conservation of matter and of impulsion, consequently nullifying the divergence of the right member and then the left member of the equation (II.9). Finally, it should be noted that the metric in this equation is a generalisation of that of MINKOWSKI.

In fact, any extension of the equation (II.9) into different energy states of matter-impulsion or into geometries of a higher order than four is a mere mind game. It would have been more relevant to seek the geometry of gravity (surfaces of equal gravitational force) in conventional space-time instead of playing with importunate (non-linear and complex) equations.

---

(1) The left member is devised on the basis of the components of the right member and the latter is referenced to the geometry of the left member.

Serious errors :

In summary, the relativists commit two unforgivable errors (of physics and mathematics), expressed as negatives :

1) Universal space cannot comprise an imaginary dimension ;
2) The dimensions of any space cannot be dependent.

Proposition :

It is a good idea to look for equations in the gravitational field of a five-dimensional architecture.

II.3 Cosmology

This supplement contains a new interpretation of HUBBLE's discovery on which our vision of contemporary cosmology is based.

HUBBLE's discovery : (see A.2.2)

The ratio (A.1) can be written : $z = H_o t$ (II.10)

where t is the time distance that separates us from the galaxy being observed.

This formula reveals two fundamental properties of extragalactic radiation :

The shift property : *every electromagnetic (or gravitational) wave doubles its period (loses half its radiation intensity) over* $\frac{1}{H_o}$ *time.*

In fact, for $f_e = 2 f_r$ one has : $z = 1$ and $t = \frac{1}{H_o}$.

This property that prevents the passage from a period $> \frac{1}{H_o}$ to a time $\leq \frac{1}{H_o}$, evokes the second property :

The minimal frequency property **:** *the maximum period of an electromagnetic wave is* $\frac{1}{H_o}$ [s].

Corollaries :

1) Since Δt is the minimum possible period of radiation, it derives from :

   * the extreme frequencies of a wave :

$$f_{max} = \frac{1}{\Delta t} \simeq 1.855 \times 10^{43} \text{ [Hz] ;}$$

$$f_{min} = H_o \simeq 2.43 \times 10^{-18} \text{ [Hz] .}$$

∗ the maximum spectral shift :

$$z_{max} = \frac{f_{max}}{f_{min}} - 1 \simeq \frac{f_{max}}{H_o} \simeq 7.635 \times 10^{60}$$

∗ the maximum causal time :

$$t_{max} = \frac{z_{max}}{H_o} \simeq \pi \times 10^{78} \text{ [s]} \qquad \text{(II.11)}$$

2) The frequency limit (H_o) characterises the minimum intensity of the energy and the temperature(1) of the vacuum :

$$\mathscr{I}_{min} = \mathbb{h} H_o \simeq 1.61 \times 10^{-51} \text{ [j/s]} \;\Rightarrow\; \theta_{min} \simeq 1.164872 \times 10^{-28} \text{ [°K]}$$

3) The frequency f_{max} radiates the maximum intensity of the energy and the temperature extreme :

$$\mathscr{I}_{max} = b \simeq 1.23 \times 10^{10} \text{ [j/s]} \;\Rightarrow\; \theta_{max} \simeq 8.9 \times 10^{32} \text{ [°K]}$$

Interpretation of the discovery :

EINSTEIN predicted that light is distorted when it passes close to a celestial body. This distortion depends on the intensity and orientation of the fields (gravitational, electrical and magnetic) through which it passes. In other words, the line of light bends in a manner that depends on the mass with which it comes into contact and its electrical charge. As we understand it, the shift measured by HUBBLE is the result of this curve. Consequently, this phenomenon is now referred to as the "curve effect". It is appropriate to indicate that the line of light only coincides with the spatial geodesic in a vacuum.

This interpretation irrevocably invalidates the expansion theory(2) and signifies the following :

---

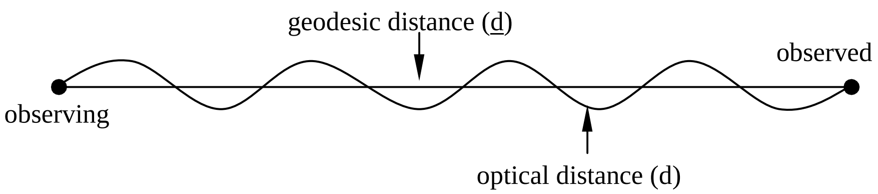
(1) The radiation from a heat source satisfies the equality of the formulae (I.8) : $\mathbb{h}\, f = \mathbb{k}\, \theta$.

(2) Whereby the density of galactic matter decreases over time, the curve of the light trajectory lessens and the shift is reduced. This does not actually accord with the experiments.

- The shift/distance proportionality (HUBBLE's observation) demonstrates the regularity of the curve along the whole length of the path of light. It is a fact that, on a very large scale, the spatio-temporal distribution of galactic matter is uniform.
- The question concerning the formation of matter and the electrical charge remains unanswered.

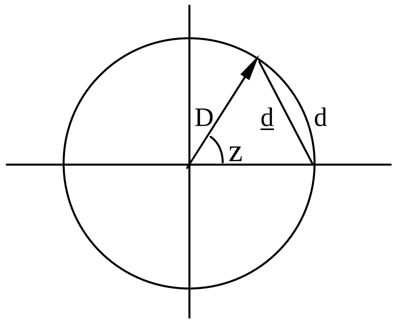
It should be mentioned that EINSTEIN's shift depreciates with every approaching/receding celestial body. It has therefore been omitted.

In any case, the curve of the optical line is virtually constant. It is manifested between two galaxies as follows :



(Fig. II.5)

The overall curvature of this line is indicated in the following circle :



(Fig. II.6)

where : z denotes the shift ;

$D \triangleq \frac{c}{H_o}$ is the radius of the curve of the trajectory ;

$\underline{d} = D\sqrt{2(1-\cos z)}$ for $z \in [0, \pi]$;

$$\text{for all } z: \ \underline{d} = 2D\left[n + \sqrt{\frac{1-\cos(z-n\pi)}{2}}\right] \ ; \ n = \left.\frac{z}{\pi}\right|_{int} \qquad \text{(II.12)}$$

Breakdown of the shift :

The imprecision of the HUBBLE constant conceals the linkage of two effects: the curvature effect and that of the incommensurable movement of the galaxies. Since the first is linear, the second is pseudo-random since the study of galactic movements over the course of history has probably been inaccurate. On the other hand, an average value for this effect can be calculated experimentally and on the basis of probability.

This being the case, the shift measured can be broken down into two quantities :

$$z = z_c + z_m \ ;$$

c and m for the curve and the movement respectively[(1)].

Implications :

In this vein, certain limitative values[(2)] of a universal nature can be detected.

Cosmic units :

By analogy with PLANCK's quantum units, HUBBLE's "cosmic units" can be defined as the maximum period and its spatial and energetic counterparts :

---

(1) By denoting in passing by $\underline{H}_o$ the precise value (in relation to the curve) of H_o, the formula (II.10) is transcribed as : $z - z_m = \underline{H}_o\, t$. This having been done, HUBBLE can be said to be responsible for the ratio : $dz = \underline{H}_o\, dt \Rightarrow z = \underline{H}_o\, t + z_m$

(2) The accuracy of these values depends on that of H_o .

$$\Delta T \ \underline{\Delta} \ \frac{1}{H_o} \ \simeq \ 4.12 \times 10^{17} \ [s] \ \simeq 1.30555 \times 10^{10} \ \text{years} \ ;$$

$$\Delta S \ = \ c \, \Delta T \simeq \ 1.236 \times 10^{26} \ [m] \ ;$$

$$\Delta E \ = \ b \, \Delta T \simeq \ 5.0649 \times 10^{27} \ [j].$$

Range of a wave :

A wave emitted at the frequency f_e travels through space for :

$$t \ = \ \frac{z}{H_o} \ = \ \frac{\frac{f_e}{H_o} - 1}{H_o} \ [s] \ \text{ hence the range : } \ d \ = \ ct \ [m] \, .$$

For $f_e >> H_o$ the result is : $d \ = \ \dfrac{cf_e}{H_o^2}$.

Great circles :

As we understand it, the only indication that reveals the curve of space (the finite nature of the universe) is causal time (t_{max}) produced by the ratio (II.11). The geodesic length of this time is calculated on the basis of (II.12) :

$$\underline{t}_{max} \ \simeq \ 2 \frac{n}{H_o} \ \simeq \ 2 \frac{z_{max}}{\pi H_o} \, .$$

The ratio between the two time periods amounts to :

$$t_{max} \ = \ \frac{\pi}{2} \ \underline{t}_{max} \, .$$

This consists of a circle with $\underline{t}_{max}$ and t_{max} as the radius and the horizon respectively.

"Grand temporal" of the universe refers to its age :

$$\mathbf{T} \simeq \ 4 \ t_{max} \ = \ 4\pi \times 10^{78} \ [s] \ \simeq 3.982 \times 10^{71} \ \text{years.}$$

The visual and energy equivalents of this circle are thus written as :

$$\mathbf{S} \ = \ c\mathbf{T} \simeq 12\pi \times 10^{86} \ [m] \ ; \ \mathbf{E} \ = \ b\mathbf{T} \simeq 1.54481992 \times 10^{89} \ [j] \, .$$

Consequently, the hypothetical radius (in the order of 6) of the cosmos is represented as :

$$\mathbf{R} \simeq \ 6 \times 10^{86} \ [m] \ \simeq \ 6.33762 \times 10^{70} \ [al] \, .$$

II.4 Electromagnetism

Any electrical or magnetic phenomenon occurring in nature is the result of an electrical charge (positive or negative). The presence of the charge induces an electrical field, its movement is known as an electrical current and the latter creates the magnetic field. The animation (or vibration) of the charge produces an influence (wave) that propagates itself at the speed of light. It is appropriate to mention here that heat(1) is a natural source of electromagnetic radiation.

A dimensional analysis of MAXWELL's equations (see A.5) produces :

For the quantities :

$[\vec{E}] = ST^{-2}$ kinetic vector ;
$[\vec{D}] = ES^{-4}T^{2}$ dynamic vector ;
$[\vec{H}] = ES^{-3}T$ dynamic vector ;
$[\vec{B}] = T^{-1}$ kinetic vector ;
$[\vec{J}] = ES^{-4}T$ dynamic vector ;
$[\vec{M}] = T^{-2}$ kinetic vector (fictitious source) ;
$[\rho] = ES^{-5}T^{2}$ dynamic quantity ;
$[\tau] = S^{-1}T^{-1}$ kinetic quantity (fictitious source).

For the equations :

[1st equation] = T^{-2} (kinetic) ;
[2nd equation] = $ES^{-4}T$ (dynamic) ;
[3rd equation] = $ES^{-5}T^{2}$ (dynamic) ;
[4th equation] = $S^{-1}T^{-1}$ (kinetic).

The modal analysis of these equations shows that :

1) All the vector fields have mode M^{-1} ;
2) The equations, like the sources, have mode M^{-2} .

---

(1) This is the only known form of pure energy.

Comment :

The electrical/magnetic action environment is transparent as to quantities and kinetic equations.

Transposition into five-dimensional space :

The expression of electrical and magnetic quantities in five-dimensional terms assumes the transparency of the environment to all of the terms indicated above. It can be spontaneously deduced that the MAXWELL equations and the structural equations are linear.

Proposition :

It is wise to rewrite the MAXWELL equations on the basis of (e; s; t).

II.5 Gravity

Just like electromagnetism, mass is at the origin of any gravitational phenomenon. Just like the electrical charge[1], the presence of mass produces a "gravitic" field, its movement is known as a gravitic current and the latter produces a "gravific" field. The vibration of mass (or the disturbance of matter) generates a gravitational wave[2] that propagates at the speed of light[3]. It should be realised that the first three quantities of gravity (m, $\vec{G}$ and γ_o) have the same dimensions (share the same natures) as their electrical analogous (q, $\vec{E}$ and ε_o). Using this line of thought, it would be plausible to establish the parallelism between electromagnetism and gravity that is illustrated by the table (Table II.1).

The gravitational equivalents of ε and μ are :

$\gamma = \gamma_o \, \gamma_r$ for gravitic permittivity ;

$\chi = \chi_o \, \chi_r$ for gravific permeability.

From this, the following is easily concluded :

$$\frac{1}{\sqrt{\gamma_o \chi_o}} = c \;\Rightarrow\; \chi_o = 9.3166 \times 10^{-27} \;[\mathrm{m^3/js^2}] .$$

Gravitational power :

Taking the example of POYNTING's vector (see A.5), the result is:

$$\vec{P}_g = \vec{G} \wedge \vec{\Omega} \;[\mathrm{W/m^2}] = \frac{1}{2}(\gamma G^2 + \chi \Omega^2)\, v \;;\; v = \frac{1}{\sqrt{\gamma\chi}}$$

In a vacuum : $v = c$ and $\vec{G} \cdot \vec{\Omega} = 0 \;\Rightarrow\; \gamma_o G^2 = \chi_o \Omega^2$.

---

(1) Charge is distinguished from mass in two ways: a charge is always associated with mass and the latter has no plus or minus sign attached to it.

(2) The term was used for the first time in 1905 in the writings of POINCARÉ.

(3) It is the speed of any non-material influence travelling through a vacuum.

Proposition :

It is fascinating to discover a system of equations for gravity that are similar to the MAXWELL equations, but transposed into five-dimensional space.

Electromagnetism			**Gravity**		
Quantity	symbol	action	Quantity	symbol	action
electricity	$\vec{E}$	electric	gravity	$\vec{G}$	gravitic
magnetism	$\vec{H}$	magnetic	gravifism	$\vec{\Omega}$	gravific
displacement	$\vec{D}$	electrical current	displacement	$\vec{\aleph}$	gravitic current
induction	$\vec{B}$	magnetic current	induction	$\vec{Q}$	gravific current
load electromagnetism	q	field electromagnetic	mass gravitation	m	field gravitational

(Table.II.1)

III. MATHEMATICAL TOOLS

Digitisation and resolution of linear differential equations

III.1 Digitisation of variables

N.B.:

This tool, used in physics, employs certain mathematical concepts in terms of its internal logic.

III.1.1 Independent Variables

General remarks :

Certain authors (non-mathematicians) confuse discontinuity with the digital, function with distribution and even a set and a series. In order to make things clear :

- *Ratio* refers to a function or distribution.
- Only the family of *applications*(1) of ratios is of interest here.
- The following definitions are applied here :
  - *Digital variable*, the monotonous set of $\mathbb{Z}$;
  - *Digital function*, a function containing digital variables ;
  - *Analogical function*, any application that uses continuous variables (admitting dx).

It should be mentioned in passing that punctual ratios such as the function $\delta(x)$ or the set $\delta(x - pX)$ of DIRAC are analogic(2).

(1) As a reminder, an application is a surjective left-handed and univocal ratio $(x \mapsto g)$.

(2) $p \in \mathbb{Z}$ (to simplify) and X is the period. According to L. SCHWARTZ, $\delta(x)$ is a function but its derivatives are distributions.

Concerning discontinuity :

Normally, discontinuity is an epithet used to describe ratios that are non-derivative in certain respects. A discontinuity such as this poses no problem for digitisation. To be more specific, discontinuity is a vertical fracture that, at one point in the variable, produces two different values for the ratio. This type of discontinuity is the subject of distributions(1), at least for physicists. For this purpose, the digitisation process needs to be adapted slightly.

Concerning the variation :

Let there be a ratio g(x) between two variables : x being independent and with uniform increase (at constant variation) and g being dependent on x. This ratio translates *the variation* of g in relation to x in three forms : analytic, graphic or digital. When g(x) is analytical and indefinitely derivable, $\frac{dg}{dx}$ is used to designate *the rate* or *speed* of this variation, through $\frac{d^2g}{dx^2}$ *the rate of the rate* of variation and so on. It should be pointed out immediately that the significant interest of a FOURIER transform is in the transcription of a differential into an algebraic term.

Dyadic spaces :

It has been established that the binary numbers 0 and 1 are the simplest, most intuitive and most natural numbers conceivable. These two fundamental states(2) of understanding constitute the basis for any digital mathematical construction and formal logic.

A number containing N binary figures represents 2^N different states. In dyadic terms, a space having N dimensions comprises 2^N points (or positions). These positions, ordered according to their decimal values, constitute a preponderant set in digital processing.

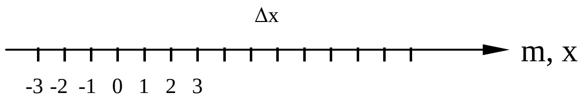
---

(1) Do not confuse the discontinuity of distribution with the continuity of their variables.

(2) The statement and its negation such as "yes" or "no", "true" or "false", etc …

Digital variation :

The variable $m \in \mathbb{Z}$, discrete by definition, does not accept analytical operators such as derivation and integration. It is consequently excluded from equations in physics. To overcome this difficulty, m is associated with the continuous axis of x in the following manner :



(Fig. III.1a)

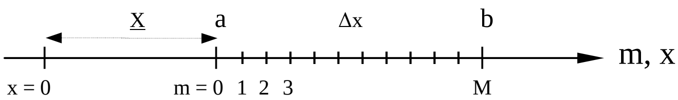
where Δx mathematically represents the unit. It is the passage from "discrete digitisation" to "analytic digitisation" dedicated to the substitution $m \Leftarrow m\Delta x$. On the other hand, the switch from continuous to digital via $x \Leftarrow m\Delta x$ is known as "digitisation". It assigns to Δx the relationship of scale between x and m. Finally, the term "associated digital variable " or more simply[1] "digital variable" is assigned to the concept $m\Delta x$.

***Variation interval*[2]:**

Digital calculation uses finite values of variables and functions. This terminates the variation interval as follows :

$$x \in [a, b] \ ; \ b - a \ \underline{\Delta} \ X$$

It is always desirable in this case to restrict digitisation to this interval and, if possible, to make the origins of x and m coincide. Where these origins are fatally separate, the substitution $x \Leftarrow \underline{X} + m\Delta x$ takes their place :



(Fig. III.1b)

---

(1) without possible risk of confusion.
(2) It corresponds physically with the field of observation, action or measurement.

Note that the property of periodicity (see A.6.2) of FOURIER digital transforms make it possible to begin the intervals of x and f at the origin of these coordinates. The result is that $\underline{X}$, like $\underline{F}$ (the equivalent of $\underline{X}$ over f) are virtually nil.

Sampling :

If digitisation is a sort of change of variable (x ← mΔx ; Δx being the indivisible step of the measurement) applied unrestrictedly to the axis of x, sampling is a technique to be applied to the interval X, one that should satisfy the dyadic requirement :

$$M = 2^N - 1 \quad ; \quad M \triangleq \left.\frac{X}{\Delta x}\right|_{int}$$

Furthermore, if M is too large, the choice of Δx must maintain the variation of the function being processed, with a single Δx that is less than the measurement threshold.

III.1.2 Dependent variables (functions)

Description :

Definition :

Like the digital variable, the standard application g(mΔx). is known as the "associated digital function" or simply the "digital function".

Criteria :

1) The analogical function and its digital version share the same analytical properties.
2) The digital function must be locally integrable[(1)].
3) Unlike the variable, the digital function may take any finite value.

---

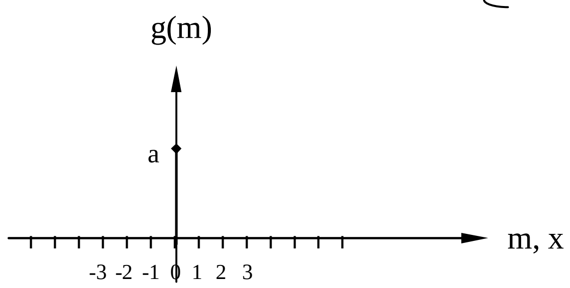
(1) Suitably digitised distributions are also integrable within the meaning attributed to them by RIEMANN.

Comment :

All analogical ratios originating in physics can be converted into digitals.

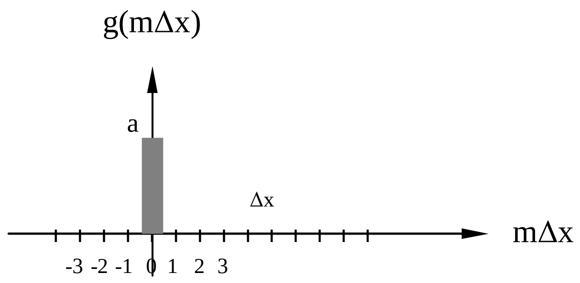
Integration :

This is the function defined by : $g(m) = \begin{cases} a & \text{for } m = 0 ; \\ 0 & \text{elsewhere.} \end{cases}$



(Fig. III.2a)

It is obvious that the Riemannian integration of this function over x is nil. This means that g(m) is independent of x and devoid of analytical meaning. In order to appropriate g(m) for integration, one must resort to the substitution $m \leftarrow m\Delta x$ so that it produces :



(Fig. III.2b)

$$\int_{-\infty}^{\infty} g(m\Delta x)\, dx = a \int_{-\frac{\Delta x}{2}}^{\frac{\Delta x}{2}} dx = a\, \Delta x \qquad \text{(III.1)}$$

Functions of DIRAC :

Analogical expression :

It is normal to use the term DIRAC "impulsion" (or measurement) to describe the function :

$$\delta(x) = \begin{cases} 1 & \text{for } x = 0 \,; \\ 0 & \text{elsewhere.} \end{cases}$$

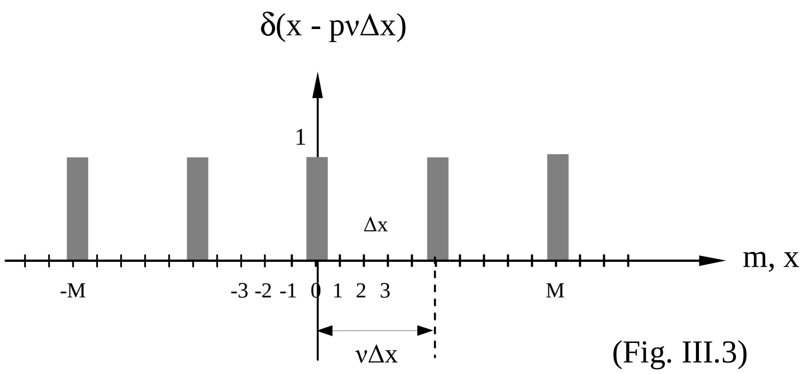
The uniform repetition (at constant interval) of this measurement generates the set :

$$\delta(x - pX) = \begin{cases} 1 & \text{for } x = pX \;;\; (p = 0, \pm 1, \pm 2, \dots, \pm\infty) \\ 0 & \text{elsewhere.} \end{cases}$$

This set, as well as the impulse, are devoid of Riemannian integration.

Digital version :

The digitisation of the x axis will verify the equality of : $X = \nu\Delta x$; ν being a whole number.



(Fig. III.3)

Integration :

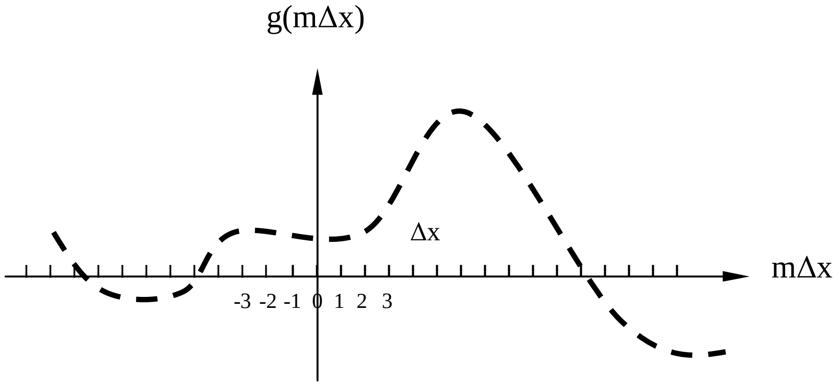
The previous set is only integrable on an interval terminating in x, [-M, M] for example, the result of the formula (III.1) is :

$$\int_{-M\Delta x}^{M\Delta x} \delta(x - p\nu\Delta x)\, dx = \sum_{p=-\frac{M}{\nu}}^{\frac{M}{\nu}} \delta(x - p\nu\Delta x)\, \Delta x = \left(1 + 2\frac{M}{\nu}\right) \Delta x$$

Typical functions :

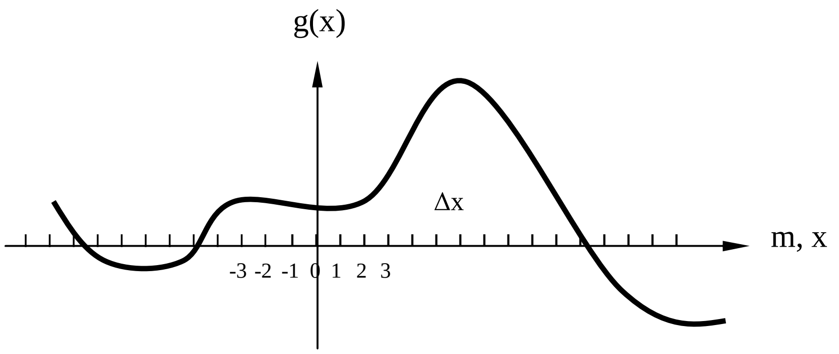
Mono-variable function :

Let there be the function :



(Fig. III.4a)

The substitution $m\Delta x \leftarrow x$ incontinently restores the analogical version :



(Fig. III.4b)

$g(m\Delta x)$ is commonly expressed as follows :

$$g(m\Delta x) \triangleq g(x)\,\delta(x - m\Delta x) \;\; ; \;\; (m = 0, \pm 1, \pm 2, \ldots, \pm\infty) \qquad \text{(III.2a)}$$

with : $\delta(x - m\Delta x) \triangleq \begin{cases} 1 & \text{for } x = m\Delta x \;\; ; \\ 0 & \text{elsewhere.} \end{cases}$

Integration :

Assuming that $g(x)$ is integrable, this produces :

$$\int_{-\infty}^{\infty} g(m\Delta x)\,dx = \int_{-\infty}^{\infty} g(x)\,\delta(x - m\Delta x)\,dx = \Delta x \sum_{m=-\infty}^{\infty} g(m\Delta x)$$

Multivariable function[1]***:***

Let us consider the function :

$$g(m_1\Delta x_1, \ldots, m_N\Delta x_N) \triangleq g(\vec{x})\,\delta(x_1 - m_1\Delta x_1)\ldots\delta(x_N - m_N\Delta x_N)$$

$$(m_1, m_2, \ldots, m_N = 0, \pm 1, \pm 2, \ldots, \pm\infty) \qquad \text{(III.2b)}$$

Integration :

$$\int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} g(\vec{x})\,\delta(x_1 - m_1\Delta x_1)\ldots\delta(x_N - m_N\Delta x_N)\,dx_1 dx_2 \ldots dx_N$$

$$= \Delta x_1 \ldots \Delta x_N \sum_{m_1=-\infty}^{\infty} \ldots \sum_{m_N=-\infty}^{\infty} g(m_1\Delta x_1, \ldots, m_N\Delta x_N)$$

Convolution :

Analogical expression :

The product of the convolution of two mono-variable functions is written as :

$$g_1(x) * g_2(x) \triangleq \int_{-\infty}^{\infty} g_1(\xi)\,g_2(x-\xi)\,d\xi$$

---

(1) It is convenient to transcribe the multivariable functions into multi-dimensional spaces.

In the case of a multivariable function, it is written :

$$g_1(\vec{x})*g_2(\vec{x}) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} g_1(\vec{\xi})\, g_2(\vec{x}-\vec{\xi})\, d\xi_1 \dots d\xi_N$$

Digital version :

Along the same lines of development, the following can be deduced :

$$g_1(m\Delta x)*g_2(m\Delta x) = \Delta x \sum_{k=-\infty}^{\infty} g_1(k\Delta x)\, g_2[(m-k)\Delta x]\ ;$$

$$(m = 0, \pm1, \pm2, \dots, \pm\infty) \qquad \text{(III.3a)}$$

$$g_1(m_1\Delta x_1, \dots, m_N\Delta x_N)*g_2(m_1\Delta x_1, \dots, m_N\Delta x_N)$$

$$= \Delta x_1 \dots \Delta x_N \sum_{k_1=-\infty}^{\infty} \dots \sum_{k_N=-\infty}^{\infty} g_1(k_1\Delta x_1, \dots, k_N\Delta x_N) \times$$

$$g_2[(m_1-k_1)\Delta x_1, \dots, (m_N-k_N)\Delta x_N]\ ;$$

$$(m_1, m_2, \dots, m_N = 0, \pm1, \pm2, \dots, \pm\infty) \qquad \text{(III.3b)}$$

Periodic functions :

Analogical expression :

A mono-variable periodic function is expressed :

$$g(x)\ \underline{\Delta}\ g(x-pX) = g_1(x)*\delta(x-pX)\ ;\ (p = 0, \pm1, \pm2, \dots, \pm\infty) \qquad \text{(III.4a)}$$

where $g_1(x)$ is the main period defined on $x \in [0, X]$.

Comment :

A periodic function is continuous if $g_1(0) = g_1(X)$ and discontinuous(1) in the opposite case.

Where the periodicity is multivariable, the result is :

$$g(\vec{x}) = g(x_1 - p_1X_1, \dots, x_N - p_NX_N)$$

$$= g_1(\vec{x})*\delta(x_1 - p_1X_1)* \dots *\delta(x_N - X_N)\ ;$$

$$(p_1, p_2 \dots, p_N = 0, \pm1, \pm2, \dots, \pm\infty) \qquad \text{(III.4b)}$$

---

(1) It is possible for $g_1(X)$ itself to be discontinuous (a distribution).

Digital version :

Since the periodic function is defined by its main period, sampling is limited to this period. As a result :

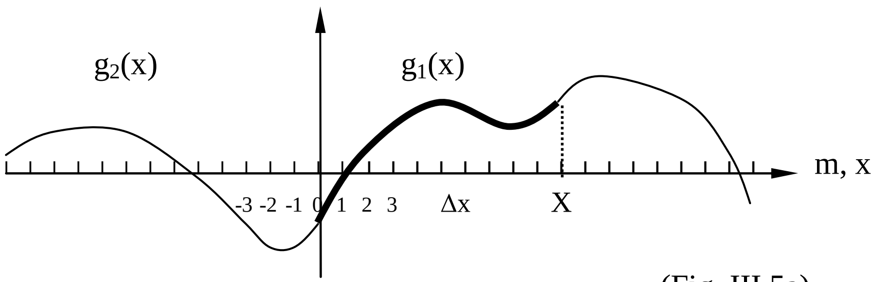
$$g_1(m\Delta x) \;=\; g_1(x)\,\delta(x - m\Delta x) \;\; ; \;\; (m = 0, 1, 2, \ldots, M) \;\; ; \qquad M\Delta x = X \qquad \text{(III.5a)}$$

$$g_1(m_1\Delta x_1, \ldots, m_N\Delta x_N) \;=\; g_1(\vec{x})\,\delta(x_1 - m_1\Delta x_1)\ldots\delta(x_N - m_N\Delta x_N)$$

$$(m_1 = 0, 1, 2, \ldots, M_1 \; ; \ldots ; \; m_N = 0, 1, 2, \ldots, M_N) \qquad \text{(III.5b)}$$

Aperiodic functions :

Let the function $g_2(x)$ be continuous and locally integrable :



(Fig. III.5a)

Let us only consider the part[(1)] $g_1(x)$ over $x \in [0, X]$ of $g_2(x)$. This can be tackled in two ways :

1) By replacing $g_2(x)$ with the above periodic function[(2)]:

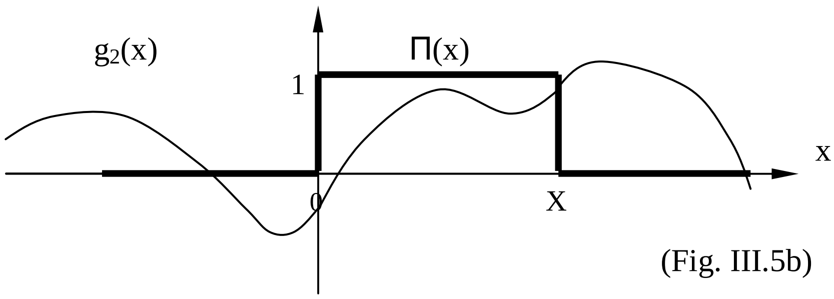
$$g(x) \;=\; g_1(x) * \delta(x - pX) \;\; ; \;\; (p = 0, \pm1, \pm2, \ldots, \pm\infty)$$

which can be digitised in accordance with (III.5a).

(1) It recalls the field of action mentioned above.
(2) This is justified by the fact that $g(x)$ reproduces $g_1(x)$ all along the axis x.

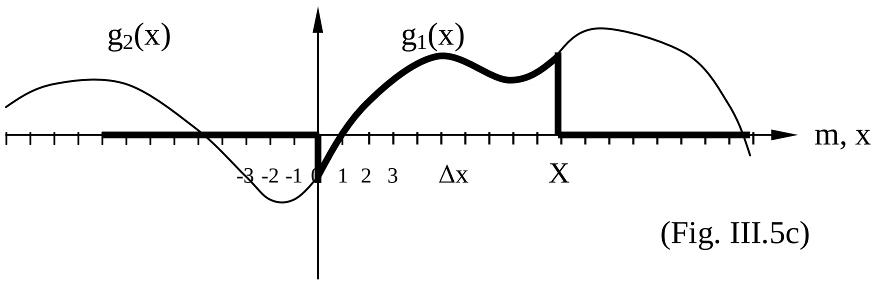
At any event, the property of (A.6.2) converts any digitised function into a periodic function having the interval of variation as the period.

2) If we multiply $g_2(x)$ by the function $\Pi(x)$:



(Fig. III.5b)

this immediately produces : $g_1(x) = \Pi(x) \times g_2(x)$



(Fig. III.5c)

Derivatives :

The following formulae are shown respectively in (A.6.1) and (A.6.2) :

$$g^{(n)}(x) = g(x)*\delta^{(n)}(x) \qquad \text{(III.6a)}$$

$$g^{(n)}(m\Delta x) = g(m\Delta x)*\delta^{(n)}(x) \ ; \ (m = 0, \pm 1, \pm 2, \ldots, \pm\infty) \qquad \text{(III.6b)}$$

III.2 Resolution of the equations

To simplify this essay, we shall restrict ourselves to mono-variable equations.

III.2.1 Algebraic equations

Equation lacking a second member :

Consider the equation :

$$g(x)(x - \xi_1)(x - \xi_2)\dots(x - \xi_K) = 0 \ ; \ g(x) \neq 0 \qquad \text{(III.7a)}$$

This permits the following solutions :

$$g(x) = a_1\delta(x - \xi_1), a_2\delta(x - \xi_2), \dots, a_K\delta(x - \xi_K) \qquad \text{(III.7b)}$$

in which a_1, a_2, …, a_K are arbitrary constants in the absence of the initial conditions. The latter must correspond to points ξ_1, ξ_2, … and be limited in number to K.

In digital form, the arbitrary constants are predictable. They are assigned the default unit where the initial conditions are lacking. In this order, the equation (III.7b) is written :

$$g(m\Delta x) = \delta(x - m_1\Delta x), \delta(x - m_2\Delta x), \dots, \delta(x - m_K\Delta x) \qquad \text{(III.7c)}$$

Two-member equation :

Let : $g(x)(x - \xi_1)(x - \xi_2)\dots(x - \xi_K) = p(x)$ (III.8a)

It can be seen that where $x = \xi_1$ or $x = \xi_2$ …, the equation loses its second member :

$$p(x) \leftarrow 0 \ \text{ and } \ g(x) = a_1\delta(x - \xi_1), a_2\delta(x - \xi_2), \dots, a_K\delta(x - \xi_K)$$

Outside the points ξ_1, ξ_2 , …, this produces :

$$g(x) = \frac{p(x)}{(x - \xi_1)(x - \xi_2)\dots(x - \xi_K)}$$

In the same way, the general solution of the equation (III.8a) produces the following result :

$$g(x) = a_1\delta(x - \xi_1),\ a_2\,\delta(x - \xi_2),\ \ldots,\ a_K\,\delta(x - \xi_K),\ \frac{p(x)}{(x-\xi_1)(x-\xi_2)\ldots(x-\xi_K)}$$

(III.8b)

The digital version of this equation is written :

$$g(m\Delta x) = \delta(x - m_1\Delta x),\ \ldots,\ \delta(x - m_K\Delta x),\ \frac{p(m\Delta x)}{(x - m_1\Delta x)\ldots(x - m_K\Delta x)}$$

(III.8c)

III.2.2 Differential equations

General form :

A linear differential equation is expressed as :

$$p_n(x)\,\frac{d^n g(x)}{dx^n} + p_{n-1}(x)\,\frac{d^{n-1}g(x)}{dx^{n-1}} + \ldots + p_1(x)\,\frac{dg(x)}{dx} + p_o(x)\,g(x) = p(x) \qquad \text{(III.9)}$$

where $p_n(x), \ldots, p_o(x)$ and $p(x)$ are known functions. It is fascinating to recall that the origin of this equation lies in physics, all of the components being locally integrable.

The digitisation of (III.9) results from the substitution :

$$x \leftarrow m\Delta x \quad ; \quad (m = 0, \pm 1, \pm 2, \ldots, \pm\infty)$$

Resolution schematic :

The equation (III.9) can be solved in four stages :

1) Digitising the equation by determining Δx and M ;
2) Applying TF (see A.6) to all its members ;
3) Separating the knowns from the unknowns ;
4) Proceeding to TF^{-1}.

Example :

For the sake of simplicity, let us make do with a 2nd degree equation :

$$p_2(x)\,\frac{d^2g(x)}{dx^2} + p_1(x)\,\frac{dg(x)}{dx} + p_o(x)\,g(x) = p(x) \qquad \text{(III.10a)}$$

The digitisation thereof for a terminated interval produces :

$$p_2(m\Delta x)\,\frac{d^2g(m\Delta x)}{dx^2} + p_1(m\Delta x)\,\frac{dg(m\Delta x)}{dx} +$$
$$p_o(m\Delta x)\,g(m\Delta x) = p(m\Delta x)\ ;\ (m = 0, 1, 2, \dots, M)$$

The application TF produces :

$$\mathbf{p}_2(w\Delta f)*\left[(j2\pi w\Delta f)^2\,\mathbf{g}(w\Delta f)\right] + \mathbf{p}_1(w\Delta f)*\left[(j2\pi w\Delta f)\,\mathbf{g}(w\Delta f)\right] +$$
$$\mathbf{p}_o(w\Delta f)*\mathbf{g}(w\Delta f) = \mathbf{p}(w\Delta f)\ ;\ (w = 0, 1, 2, \dots, W) \ \Rightarrow$$

$$\Delta f \sum_{k=0}^{W} \mathbf{g}[(w-k)\Delta f]\left\{\mathbf{p}_2(k\Delta f)\,[j2\pi(w-k)\Delta f]^2 + \mathbf{p}_1(k\Delta f)\,[j2\pi(w-k)\Delta f]\right.$$
$$\left. + \mathbf{p}_o(k\Delta f)\right\} = \mathbf{p}(w\Delta f)\ ;\ (w = 0, 1, 2, \dots, W) \qquad \text{(III.10b)}$$

The periodicity of $\mathbf{g}(w)$ in W enables the substitution :

$$\mathbf{g}[(w-k)\Delta f] \Leftarrow \mathbf{g}[(W+w-k)\Delta f] \quad \text{where } w < k$$

As a result, the variation of w in the equation (III.10b) generates an algebraic system, from $W + 1$ linear equations, containing unknowns(1) W of the type $\mathbf{g}(w\Delta f)$.

The resolution of the latter results in the functions $\mathbf{g}(w\Delta f)$ which with the help of TF^{-1}, serve to reconstruct the function $g(m\Delta x)$.

Example of calculation :

$$\text{Let } \Delta f\left\{\mathbf{p}_2(k\Delta f)\,[j2\pi(w-k)\Delta f]^2 + \mathbf{P}_1(k\Delta f)\,[j2\pi(w-k)\Delta f] + \right.$$
$$\left.\mathbf{p}_o(k\Delta f)\right\} = \boldsymbol{\phi}(w\Delta f, k\Delta f)$$

---

(1) Because the continuity of $\mathbf{g}(f)$ implies : $\mathbf{g}(0) = \mathbf{g}(W)$.

The following is deduced for :

<u>w = 0 :</u>

$$\mathbf{g}(0)\,\phi(0, 0) + \mathbf{g}[(W - 1)\Delta f]\,\phi(0, \Delta f) + \ldots + \mathbf{g}(0)\,\phi(0, W\Delta f) = \mathbf{p}(0)$$

<u>w = 1 :</u>

$$\mathbf{g}(\Delta f)\,\phi(\Delta f, 0) + \mathbf{g}(0)\,\phi(\Delta f, \Delta f) + \ldots + \mathbf{g}(\Delta f)\,\phi(\Delta f, W\Delta f) = \mathbf{p}(\Delta f)$$

.
.
.

<u>w = W :</u>

$$\mathbf{g}(W\Delta f)\,\phi(W\Delta f, 0) + \mathbf{g}[(W - 1)\Delta f]\,\phi(W\Delta f, \Delta f) + \ldots + \mathbf{g}(0)\,\phi(W\Delta f, W\Delta f) = \mathbf{p}(W\Delta f)$$

Remembering that $\mathbf{g}(W\Delta f) = \mathbf{g}(0)$ and $\mathbf{p}(W\Delta f) = \mathbf{p}(0)$.

This system is finally shown as :

$$\left.\begin{array}{l}
c_{00}\,\mathbf{g}(0) + c_{01}\,\mathbf{g}(\Delta f) + \ldots + c_{0(W-1)}\,\mathbf{g}[(W - 1)\Delta f] = \mathbf{p}(0) \\
c_{10}\,\mathbf{g}(0) + c_{11}\,\mathbf{g}(\Delta f) + \ldots + c_{1(W-1)}\,\mathbf{g}[(W - 1)\Delta f] = \mathbf{p}(\Delta f) \\
\quad . \\
\quad . \\
\quad . \\
c_{(W-1)0}\,\mathbf{g}(0) + c_{(W-1)1}\,\mathbf{g}(\Delta f) + \ldots + \\
\qquad\qquad c_{(W-1)(W-1)}\,\mathbf{g}[(W - 1)\Delta f] = \mathbf{p}[(W - 1)\Delta f] \\
c_{W0}\,\mathbf{g}(0) + c_{W1}\,\mathbf{g}(\Delta f) + \ldots + c_{WW}\,\mathbf{g}[(W - 1)\Delta f] = \mathbf{p}(0)
\end{array}\right\} \quad \text{(III.10c)}$$

in which c_{wi} are numbers.

Special case :

For $p(m\Delta x) = 0$, the system (III.10c) accepts non-trivial solutions since two, and only two, of its equations (the first and last) are linearly dependent.

CONCLUSION

In this memorandum :

- Universal space is described.
- The spatial dimensions and those of measurement are unified.
- Differential equations of nature are rendered linear.
- The effects of acceleration, the distortion of light and the HUBBLE shift are interpreted originally.
- The Theory of Relativity and the BIG BANG Theory are disproved.
- An own method of digitisation and an approach for solving linear differential equations are devised.

It is now up to young scientists :

- to rewrite all the equations on the basis of (e; s; t) ;
- to establish authentic equations for gravity ;
- to study the cosmos without seeking an origin for it ;
- to refine the mathematical tool and develop an algorithm for solving multivariable linear differential equations.

Finally, it would be wise to permanently curb an enthusiasm for generalisation and the desire to know everything.

APPENDICES

A.1 Systems of measurement

A.1.1 International system of units

This system[1], completed in 1960 and dubbed SI, consists of 7 fundamental quantities and replaces all the previous systems : CGS, MKSA, MKSC, etc … (see Table A.1)

Definitions :

Second : (1967)

The second lasts for 9,192,631,770 periods of radiation corresponding to the transition between the two hyperfine levels of the fundamental state of the atom of caesium 133.

Metre :

(1960) The metre is a measurement of length equal to 1,650,763.73 wavelengths in a radiation vacuum corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the atom of krypton 86.

(1983) The metre is the length travelled by light in a vacuum for a period of $\dfrac{1}{299{,}792{,}458}$ [s].

Candela : (1979)

The candela is the luminous intensity, in a given direction, of a source emitting monochromatic radiation at a frequency of 540×10^{12} [Hz], which energy intensity in this direction is $\dfrac{1}{683}$ [W/sr].

---

(1) It has constantly evolved and was more or less "relativised" in 1982 by abandoning LENGTH as a fundamental quantity in favour of the speed of light.

Quantity	Symbol	Origin	Dimension	Unit	Unit symbol
Length	ℓ	Kinetic	L	metre	m
Time	t	Kinetic	T	second	s
Mass	m	Dynamic	M	kilogram	kg
Intensity of electric current	I_e	Electrical	I	ampere	A
Temperature	T	Thermodynamic	Θ	kelvin	K
Intensity of light	I_v	Optic		candela	cd
Quantity of matter	Q_m	Chemical		mole	mol

(Table A.1)

Mole : (1971)

A mole is a quantity of matter in a system containing as many elementary entities as there are atoms in .012 [kg] of carbon 12.

Comment :

It would have been more relevant to calibrate the metre as a distance covered by light passing through a vacuum for a period lasting $\frac{1}{3} \times 10^{-8}$ [s] representing, very precisely, 30.6421059 periods of the radiation referenced to the second.

A.1.2 System proposed

Our system is based on the new fundamental quantities, their dimensions, modes and units. The following tables show the old fundamental quantities (mass, electrical charge and temperature), as an indication, a few derived quantities and finally the principal constants discussed in this essay.

Quantities : (see Table A.2)

Constants : (see Table A.3)

Type	Quantity	Symbol	Dimension	Mode	Unit	Former dimension
Fundamental quantities						
	Energy	**e**	**E**	**M**	**j**	S^2MT^{-2}
	Length	**s**	**S**	**M**	**m**	S
	Time	**t**	**T**	**M**	**s**	T
Former fundamental quantities						
	Mass	m	$ES^{-2}T^2$	M	Kg	M
	Electrical charge	q	$ES^{-2}T^2$	M	C	Q
	Temperature	θ	T^{-1}	M^{-1}	K	Θ
A few derivative quantities						
	Quantity of movement	p	$ES^{-1}T$	M	js/m	MST^{-1}
	Density of energy (longitudinal)	e_ℓ	ES^{-1}	M^0	j/m	SMT^{-2}
	(Surface)	e_s	ES^{-2}	M^{-1}	j/m^2	MT^{-2}
	(Volume)	e_v	ES^{-3}	M^{-2}	j/m^3	$S^{-1}MT^{-2}$
	Power	P	AND^{-1}	M^0	j/s = W	S^2MT^{-3}
	Heat	W	E	M	j	S^2MT^{-2}
	Entropy, Action	S	AND	M^2	js	S^2MT^{-1}
	Force	F	ES^{-1}	M^0	N	SMT^{-2}

	Intensity of gravitic field	G	ST^{-2}	M^{-1}	m/s^2	ST^{-2}
	Time frequency	f	T^{-1}	M^{-1}	Hz	T^{-1}
	Density of the volume of the electrical charge	ρ	$ES^{-5}T^2$	M^{-2}	C/m^3	QS^{-3}
	Intensity of the electric current	I	$ES^{-2}T$	M^0	A	QT^{-1}
	Surface density of the electric current	J	$ES^{-4}T$	M^{-2}	A/m^2	$QS^{-2}T^{-1}$
	Intensity of the electrical field	E	ST^{-2}	M^{-1}	$m/s^2 = V/m$	$SMT^{-2}Q^{-1}$
	Electrical displacement	D	$ES^{-4}T^2$	M^{-1}	$Js^2/m^4 = C/m^2$	QS^{-2}
	Intensity of the magnetic field	H	$ES^{-3}T$	M^{-1}	A/m	$S^{-1}T^{-1}Q$
	Magnetic induction	B	T^{-1}	M^{-1}	1/s	$MT^{-1}Q^{-1}$
	Number of waves	k_o	S^{-1}	M^{-1}	1/m	S^{-1}

(Table A.2)

Type	Name	Symbol	Dimension	Mode	Unit	Former Dimension
Fundamental constants						
	NEWTON's constants	$\boldsymbol{\gamma_o}$	$ES^{-5}T^{4}$	M^{0}	js^{4}/m^{5}	$S^{-3}MT^{2}$
	Dielectric permittivity of the vacuum	$\boldsymbol{\varepsilon_o}$	$ES^{-5}T^{4}$	M^{0}	$Js^{4}/m^{5} = F/m$	$S^{-3}T^{2}M^{-1}Q^{2}$
	Magnetic permeability of the vacuum	$\boldsymbol{\mu_o}$	$E^{-1}S^{3}T^{-2}$	M^{0}	$m^{3}/js^{2} = H/m$	SMQ^{-2}
Fundamental measures						
	PLANCK's measurement (modified)	$\mathbb{h}$	E	M	j	$S^{2}MT^{-2}$
	HUBBLE measurement	$\mathbf{H}_o$	T^{-1}	M^{-1}	Hz	T^{-1}
Other constants						
	Speed of light	c	ST^{-1}	M^{0}	m/s	ST^{-1}
	BOLTZMANN measurement (modified)	$\mathbb{k}$	E	M	j	$S^{2}MT^{-2}$

(Table A.3)

A.2 Frequency shifts

The subject of this appendix is two shifts, the shift in movement detected by DOPPLER and the shift in curve revealed by HUBBLE.

A.2.1 DOPPLER shift

DOPPLER discovered that a wave (an electromagnetic wave in particular) emitted (or received) by a moving object loses (or gains) energy in the form of a positive or negative spectral shift, depending on its direction of movement[1] and its speed.

In a monochrome electromagnetic transmission between two remote units, one being in motion in relation to the other, the shift between the frequency emitted (f_e) and that received (f_r) will be :

$$\Delta f \,\underline{\Delta}\, \left| f_e - f_r \right| = f_e \frac{v}{c} \;\Rightarrow\; \frac{\left| f_e - f_r \right|}{f_e} = \left| 1 - \frac{f_r}{f_e} \right| = \frac{v}{c}$$

As a result :

$$f_r = f_e \pm f_e \frac{v}{c} = f_e \left(1 \pm \frac{v}{c} \right) ;$$

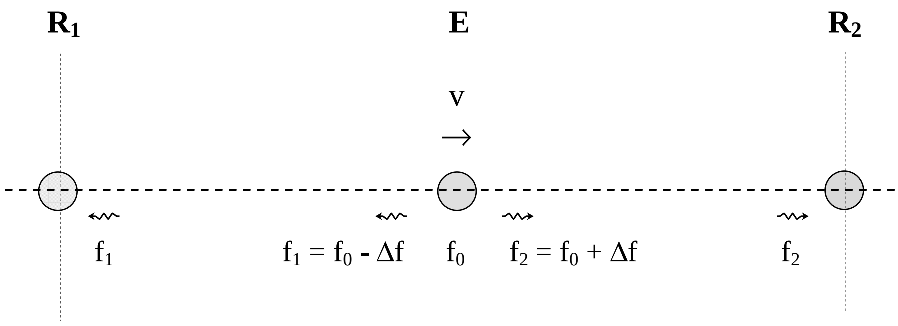
+ and $-$ indicates the direction of movement (advancing movement or receding movement).

In order to understand this better, let us consider a transmission alternating between three units, one mobile and two fixed, in relation to the scene. Then let us note the shift in the following two cases :

---

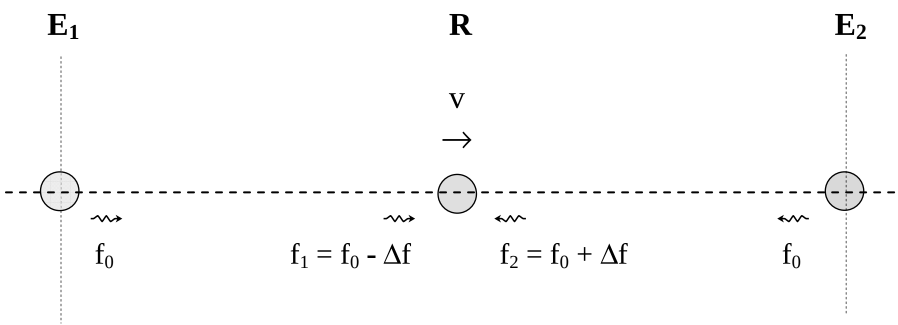
(1) Whether free or dependent.

1) The transmitter (**E**) is mobile and the receivers (**R**$_1$ and **R**$_2$) are fixed :



(Fig. A.1a)

2) The receiver (**R**) is mobile and the transmitters (**E**$_1$ and **E**$_2$) are fixed :



(Fig. A.1b)

A.2.2 The HUBBLE shift

HUBBLE established[1] that light emitted from a distant galaxy suffers a loss of radiation energy proportional to its distance. This loss, translated into a spectral shift towards red, is isotropic and complies with the ratio :

$$\frac{cz}{d} = H_o \qquad \text{(A.1)}$$

with :

d being the distance between ourselves and the galaxy observed ;

$z \triangleq \frac{f_e}{f_r} - 1$ being the measured shift; f_e and f_r designating the frequencies emitted and received respectively ;

H_o is the famous HUBBLE constant.

It is crucial to remember that d and f_e are determined independently[2] of the HUBBLE experiment.

Interpretation :

HUBBLE was obviously the first to interpret his exploration :

The shift of the light is a DOPPLER effect, shown as :

$$z = \frac{v}{c} \Rightarrow v = H_o d$$

This first degree interpretation caused other scientists to produce the following scenario :

(1) Work completed in 1928 and published a year later.
(2) This certainly justifies HUBBLE's contribution.

The shift proportional to distance is the result of the divergent recession (accelerated receding motion) of the galaxies. This assumes that in the past, the galaxies did not move as fast and were closer together. By reviewing the history of the universe, it can be seen that in a distant era the heavenly bodies were clustered together and even crushed one against the other. The cosmos thus imploded into a single body subject to its own gravity that reduced it to PLANCK's dimensions. This critical and unstable state (known as singular) of the universe soon exploded, giving birth to everything that followed. This mysterious "origin" of the world was dubbed the "BIG BANG" by HOYLE(1) in 1948. From then on, astrophysicists worked tirelessly, but without significant result, to design a mathematical model that would fit this strange singularity.

As we understand it, this scenario(2) is too speculative because we are unaware of any probable accelerator of the galaxies and if it were the case, their initial velocities would have been nil. This undeniably refutes the initial hypothesis of the BIG BANG theory.

At any event, HUBBLE's work and especially his discovery of the shift are of vital importance for cosmology (see II.3).

Comment :

Contrary to what the literature teaches us, the DOPPLER and HUBBLE shifts are not defined in the same way :

$$z_D \triangleq \frac{\Delta f}{f_e} \quad \text{whereas} \quad z_H \triangleq \frac{\Delta\lambda}{\lambda_e} = \frac{\Delta f}{f_r}$$

---

(1) The founders of the theory are FRIEDMAN, LEMAÎTRE and GAMOW.

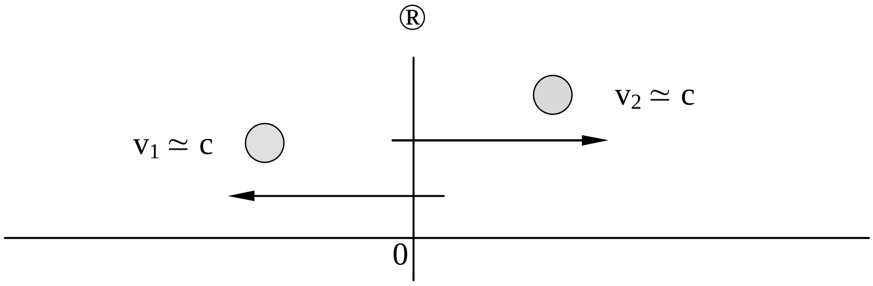
(2) Rectified by the relativists who recommend the expansionist space-time model.

A.3 Effect of acceleration

A.3.1 Independent velocities

Independent velocities do not produce any mechanical effect. They simply obey the traditional vectorial rules of composition and accumulate speeds greater than c but always less than 2c.

As an illustration, suppose two bodies were receding from 0 at a speed close to c.



(Fig. A.2a)

The relative speed (of one of the bodies in relation to the other) at which it recedes is close to 2c. It is worth knowing that mobility, at a speed greater than c, crosses the event horizon and prevents any interaction.

At the same time, the speed of the advance would be written :

$$v = v_1 + v_2 \simeq 2c$$

(Fig. A.2b)

A.3.2 Dynamic applications

Impulse :

$$J = \int_0^t m_o \frac{d}{dt}\left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right) dt = m_o \int_0^v d\left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right) = m_o \frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$$

Kinetic energy :

$$E_c = \int_0^s m_o \frac{d}{dt}\left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right) ds = m_o \int_0^v v\, d\left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$= m_o \left| \frac{v^2}{\sqrt{1-\frac{v^2}{c^2}}} - \int \frac{v}{\sqrt{1-\frac{v^2}{c^2}}} dv \right|_0^v$$

$$= m_o \left| \frac{v^2}{\sqrt{1-\frac{v^2}{c^2}}} + c^2 \sqrt{1-\frac{v^2}{c^2}} \right|_0^v = m_o \left. \frac{v^2 + c^2\left(1-\frac{v^2}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} \right|_0^v$$

$$= m_o \left. \frac{c^2}{\sqrt{1-\frac{v^2}{c^2}}} \right|_0^v = m_o c^2 \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

A.4 Relativity

Relativity is the study of natural movements assuming the elasticity of space-time and the interdependence of primary natures.

A.4.1 Natural movements

Modern physics distinguishes between four forms of natural movement :

1) nil movement,
2) straight movement,
3) movement of falling bodies,
4) gravitational movement of the planets.

The first two were considered by GALILEO to be inert[(1)] and equivalent (of the same essence). NEWTON revealed the identity of the last two (falling bodies and gravitational movement) and maintained the difference between inertial (kinetic) and gravitational (dynamic) movement. EINSTEIN stipulated in his Special Theory of Relativity that inertial movement describes the geodesics of space-time and claimed, in his General Theory of Relativity, that there was an equivalence between inertia and gravity : *all natural movement is inertial*. He explained this as follows : *the planets gravitate on the geodesics of space without being subject to any force. They are effectively free bodies and their movement is inertial.*

---

(1) Galilean principle of inertia : *"In relation to any free physical system used as a reference, any other free system remains at rest or is animated by uniform movement in a straight line"*.

A.4.2 Restricted relativity

Physical aspect :

Restricted relativity relates to straight-line, uniform, kinetic movement. It is based on two principles of invariance and two hypotheses.

Principles :

1) *"The laws of nature all take the same form in all the inertia systems of reference".*
2) *"The speed of light is an absolute constant independent of any reference point ".*

Hypothesis :

1) LORENTZ's Contraction : *"Lengths contract in the direction of their movement".*
2) *"Inertial movements describe the geodesics of space-time".*

Implications :

1) With acceleration, time dilates and mass increases.
2) The laws of nature *must be* invariable in the LORENTZ group.
3) *The equations of the geodesics of space-time are the equations of Galilean movement*.

Formalism : (1905)

Dilatation of time : $t_2 = t_1\sqrt{1 - \frac{v^2}{c^2}}$

Increase in mass : $m_2 = \frac{m_1}{\sqrt{1 - \frac{v^2}{c^2}}}$

MINKOWSKI's metric : $ds^2 = dx^2 + dy^2 + dz^2 + j^2c^2dt^2$

Galilean law of inertia : $a^k = \dfrac{dv^k}{dt} = \dfrac{d^2x^k}{dt^2} = 0$; (A.2)

$x^k(t)$ is the spatial trajectory of a free particle of a mass that is not nil ;

t is the time parameter.

A.4.3 General Relativity

Physics aspect :

General relativity applies to the movement of heavenly bodies. Its author maintains the principles and hypotheses of special relativity and adds two other principles and a new hypothesis.

Principles :

1) The MACH Principle : *the geometry of space is determined by its material content.*
2) Principle of covariance : (tensorial version of invariance)

   The tensorial expression of the laws of physics must be in covariant form.

Hypothesis : the curve of space

The curved mass in surrounding space (matter-space action). Consequently, EINSTEIN ignored the gravitational forces and advanced his "principle of equivalence" : the planetary movements are also inertial.

Implications :

1) A strong gravitational field slows the clocks and shifts the spectrum of light.
2) *The geodesic equations of space-time are the equations of the natural movements*.

Formalism : (1915)

Metric : $ds^2 = g_{\mu\nu}(x^1, x^2, x^3, x^4)\, dx^{\mu} dx^{\nu}$; $x^4 = jct$

Law of inertia : generalisation of the equation (A.2)

$$a^k = \frac{\delta v^k}{\delta t} = \frac{d^2 x^k}{dt^2} + \left\{ {k \atop pq} \right\} \frac{dx^p}{dt} \frac{dx^q}{dt} = 0$$

Gravitational field equation :

EINSTEIN translated the MACH's principle into a tensorial equation describing the geometry of the content (forged by gravity) on the left and the content (matter and impulse) on the right as follows :

$$\frac{1}{2} g_{\mu\nu} R - R_{\mu\nu} = kT_{\mu\nu} \;;\; (\mu, \nu = 1, 2, 3, 4 \text{ dimensions}) \qquad \text{(A.3)}$$

This equation is deduced from the principle of the least HAMILTON action transcribed by HILBERT for the infinitesimal variations $g_{\mu\nu}(x) \Leftarrow g_{\mu\nu}(x) + \delta g_{\mu\nu}(x)$ that are cancelled at the frontier of the area of integration, in the form :

$$\delta(S_g + S_m) = 0 \;;\; S_g = \int L_g \sqrt{g}\, d^4x \;;\; S_m = \int L_m \sqrt{g}\, d^4x$$

where : S_g and S_m are respectively the action of the gravitational field and that of matter ;

$\sqrt{g}\, d^4x$ is the element of volume.

A hypothesis produced by EINSTEIN consists in admitting that the L_g scalar should only be expressed as a function of $g_{\mu\nu}$ and its derivatives. The only suitable scalar is the curve R in a Riemannian space. This results in :

$$S_g = \kappa_g \int R \sqrt{g}\, d^4x \;;\; \kappa_g \text{ is a constant.}$$

The variation (δS_g) of this ratio leads to the left member of the equation (A.3) while the right member is drawn from the proposition :

$$T^{\mu\nu} \underline{\Delta} \; \delta S_m$$

The terms contained in the equation (A.3) are expressed as follows [(1)] :

$g_{\mu\nu}$ are components of the metric, functions of the coordinates (s; t) ;

$R_{\mu\nu}$ is the tensor of the geometric RICCI curve, its components are functions of (s; t) ;

R is the scalar of the Riemannian curve area, a function of (s; t) ;

k is a constant of the Newtonian limit of the equation (A.3), determined in 1916 by SCHWARZSCHILD :

$$k = \frac{8\pi G}{c^4} \ ; \ G = \frac{1}{4\pi\gamma_o} \text{ (gravity constant) ;}$$

$T_{\mu\nu}$ is the matter-impulse tensor, fundamental in the mechanics of a continuous medium; its components at each point of space-time, are functions of the pressure (p) and density (ρ) of this medium.

In order to close space and *enable the possibility of a virtually static distribution of matter*, in 1917. EINSTEIN (in an analogy to POISSON's equation) added to the left member of the equation (A.3) the term ($- \Lambda\, g_{\mu\nu}$; Λ being *a cosmological constant*). He revised his proposition in 1931.

Expressed in this way, the equation (A.3) takes the form :

$$\frac{1}{2} g_{\mu\nu} R - R_{\mu\nu} - \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad \text{(A.4)}$$

In 1928, E. CARTAN demonstrated the general nature of the left member of this equation.

(1) We have deliberately avoided relativist vocabulary such as "geometry of space" or "impulse-energy" metric, etc.

General properties :

1) The equation (A.4) is a local equation, valid for a complex four-dimensional geometry in which the fourth dimension is imaginary.
2) The linearity of the equation (A.4) is governed by the intensity of the gravitational field. It is linear, in the first approximation, when the fields are weak and non-linear in the opposite case(1).
3) All of EINSTEIN's tensors are symmetrical and with nil divergence(2).

Comment :

1) The purpose of the equation (A.4) is to determine the geometry of the gravitational action from its material sources.
2) The dimension(3) of the equation (A.4) is $[S^{-2}]$ thus :

$$[g_{\mu\nu}] = S^0 \; ; \; [R] = [R_{\mu\nu}] = [\Lambda] = S^{-2} \; ;$$

$$\left[\frac{8\pi G}{c^4}\right] = E^{-1}S \; ; \; [T_{\mu\nu}] = ES^{-3}$$

From this the measurements : $[R] = [R_{\mu\nu}] = [\Lambda] = 1/m^2$ can be deduced ;

$$\left[\frac{8\pi G}{c^4}\right] = m/j \; ; \; [T_{\mu\nu}] = j/m^3$$

From the outset, it is significant to note that :

- R, like Λ, represents a two-dimensional curve ;
- $\frac{8\pi G}{c^4}$ it is of the same nature as the constant "a" (see §1.2.3) ;

---

(1) since the strong fields do not allow for the principle of superimposition.
(2) The left tensor verifies the BIANCHI identity, contracted on the pairs of indices pq and $r\alpha$, the right-hand one satisfying the laws of conservation of matter and of impulse.
(3) independently of any form of expression.

- $T_{\mu\nu}$ expresses a volumic density of energy.

Unified fields :

In 1925, EINSTEIN attempted to unify the gravitational and electromagnetic fields. For this purpose, he considered the principle of variation :

$$\delta \int g^{\nu\mu} R_{\mu\nu} \sqrt{g}\ d^4x = 0$$

and the non-symmetry of the fundamental tensor as well as of the connection $\Gamma^{\lambda}_{\nu\mu}$ (CHRISTOFFEL symbol of the 2nd kind) linked thereto. This initiative enabled him to break down the fundamental tensor into two parts, symmetrical and anti-symmetrical, corresponding respectively to the gravitational and the electromagnetic fields. In 1945, he obtained the ratio :

$$g_{\mu\nu,\rho} \ \underline{\Delta}\ \frac{\partial g_{\mu\nu}}{\partial x^{\rho}} - g_{\lambda\nu}\ \Gamma^{\lambda}_{\mu\rho} - g_{\mu\lambda}\ \Gamma^{\lambda}_{\rho\nu} = 0$$

which later became a consequence of the principle of variation.

Finally, he deduced the equations :

$$g_{\mu\nu,\rho} = 0 \ ; \ \Gamma_{\mu} = 0 \ ; \ \underline{R}_{\mu\nu} = 0 \ ; \ \underline{\underline{R}}_{\mu\nu,\lambda} + \underline{\underline{R}}_{\lambda\mu,\nu} + \underline{\underline{R}}_{\nu\lambda,\mu} = 0$$

where : $\Gamma_{\mu} \ \underline{\Delta}\ \frac{1}{2}\left(\Gamma^{\lambda}_{\mu\lambda} - \Gamma^{\lambda}_{\lambda\mu}\right)$ is a quadri-vector cancelling itself out identically in the Riemannian case ;

$\underline{R}$ and $\underline{\underline{R}}$ are the parts, symmetrical and anti-symmetrical, of $R_{\mu\nu}$.

In fact, this last equation is only a shortened version of the BIANCHI identity.

In any case, no attempts to generalise the Riemannian connection ever produces MAXWELL equations for the free field.

A.4.4 Memoranda

Concepts in physics :

Free body : this is a body that is not subject to the action of any force (including the state of balance, the nil sum of forces).

Invariance :

A quantity in physics (Φ) is known as an "invariant" if it has the same value in all the systems of reference :

$$\Phi(s; t) = \Phi'(s'; t')$$

HAMILTON's principle :

A particle moves in such a way that $\int_{t_1}^{t_2} L\,dt$ *is an extremum ;*

L being the Lagrangian.

Mathematical concepts :

A few elementary properties of tensors :

1) The properties of symmetry (anti-symmetry) are invariable to any change of marker.
2) Any tensor may be expressed as the sum of two tensors, one being symmetrical and the other anti-symmetrical for a pair of co-variant or contra-variant indices.
3) Covariance describes the variation of the functions of coordinates, independently of any reference.

Conjugated or reciprocated tensors :

If $g = |g_{pq}|$ is the determinant of the elements g_{pq} and if we assume that $g \neq 0$, the conjugated (or reciprocal) tensor of g_{pq} is defined by :

$$g^{pq} \triangleq \frac{\text{cofactor of } g_{pq}}{g} \; ; \; g^{pq} g_{rq} = \delta^p_r$$

CHRISTOFFEL Symbols :

$$1^{st}\text{ kind : } [pq\,,r] = \frac{1}{2}\left[\frac{\partial g_{pr}}{\partial x^q} + \frac{\partial g_{qr}}{\partial x^p} - \frac{\partial g_{pq}}{\partial x^r}\right]$$

$$2^{nd}\text{ kind : } \left\{ {s \atop pq} \right\} = g^{sr}\,[pq\,,r]$$

Remember that :

$$[pq\,,r] = [qp\,,r]\ ;\ \left\{ {s \atop pq} \right\} = \left\{ {s \atop qp} \right\}\ ;\ [pq\,,r] = g_{rs}\left\{ {s \atop pq} \right\}$$

In particular, if $\left\{ {s \atop pq} \right\}$ is a tensor, it is standard to write it Γ^s_{pq}. The CHRISTOFFEL symbols are cancelled out in the orthogonal systems.

Geodesics :

Geodesics is the topology of space. "Geodesics" are the lines that satisfy the differential equation :

$$\frac{d^2x^r}{ds^2} + \left\{ {r \atop pq} \right\} \frac{dx^p}{ds}\frac{dx^q}{ds} = 0$$

where s is the distance between two points on the curvilinear abscissa x^r of Riemannian space.

Metric : (quadratic fundamental)

The metric is the elementary measurement on the geodesic :

$$ds^2 = g_{pq}\,dx^p\,dx^q\ ;\ (p, q = 1, 2, \dots, N \text{ dimensions})$$

Properties :

1) Symmetry of the components : $g_{pq} = g_{qp}$
2) Non-nil determinant : $|g_{pq}| \underline{\Delta}\ g \neq 0$
3) Invariance : the metric is invariable to a change of scale marker.

In particular, if $g_{pq} = 0$ for $p \neq q$ then the geometry is Euclidian and the metric is reduced to :

$$ds^2 = (dx^i)^2\ ;\ (i = 1, \dots, N)$$

Covariant derivative :

The covariant derivative of a tensor A_p in relation to x^q is :

$$A_{p,q} \triangleq \frac{\partial A_p}{\partial x^q} - \left\{ {s \atop pq} \right\} A_s$$

Similarly, the covariant derivative of a tensor A^p in relation to x^q is :

$$A^p_{,q} \triangleq \frac{\partial A^p}{\partial x^q} + \left\{ {p \atop qs} \right\} A^s$$

Note that :

- the covariant derivatives of g^{pq} , g_{rq} and δ^p_r are nil ;
- in orthogonal systems, the covariant derivatives are the usual partial derivatives.

Divergence :

The divergence of A^p is the contraction of its covariant derivative in relation to x^q :

$$\operatorname{div} A^p \triangleq A^p_{,p} = \frac{1}{\sqrt{g}} \frac{\partial (\sqrt{g}\, A^k)}{\partial x^k} \quad ;$$

$\sqrt{g}$ is a tensorial density (pseudo-tensor of a unit weight).

Intrinsic** (or absolute) **derivative :

The absolute derivative of a vector A_p along the length of a curve $x^q(t)$ is defined as the contracted product of the covariant derivative of A_p and of $\frac{dx^q}{dt}$:

$$\frac{\delta A_p}{\delta t} \triangleq A_{p,q} \frac{dx^q}{dt} = \frac{dA_p}{dt} - \left\{ {r \atop pq} \right\} A_r \frac{dx^q}{dt}$$

Similarly :

$$\frac{\delta A^p}{\delta t} \triangleq A^p_{,q} \frac{dx^q}{dt} = \frac{dA^p}{dt} + \left\{ {p \atop qr} \right\} A^r \frac{dx^q}{dt}$$

It should be emphasised that the intrinsic derivatives of g^{pq} , g_{rq} and δ^p_r nil (RICCI’s Theorem).

Element of volume : $dv = \sqrt{g}\, dx^1 dx^2 \ldots dx^N$

It is invariable for any system of coordinates.

Tensor of curvature : (RIEMANN - CHRISTOFFEL)

$$R^{\alpha}_{pqr} \triangleq \frac{\partial \Gamma^{\alpha}_{pq}}{\partial x^{r}} - \frac{\partial \Gamma^{\alpha}_{pr}}{\partial x^{q}} + \Gamma^{\alpha}_{\beta r}\,\Gamma^{\beta}_{pq} - \Gamma^{\alpha}_{\beta q}\,\Gamma^{\beta}_{pr} \neq 0$$

Through the contraction of this tensor, RICCI's symmetrical tensor is obtained :

$$R_{pq} = R^{\alpha}_{pq\alpha}$$

The contraction of the latter provides the scalar of curvature :

$$R = R^{\alpha}_{\alpha}$$

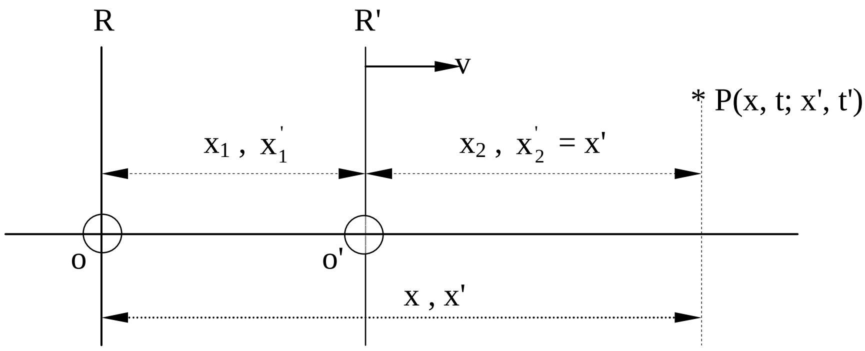
These tensors are nil in Euclidian geometry.

BIANCHI's Identity : $R^{\alpha}_{pqr,\beta} + R^{\alpha}_{p\beta q,r} + R^{\alpha}_{pr\beta,q} \equiv 0$

This is valid for any symmetrical connection.

A.4.5 LORENTZ Transformations

If we return to (§ II.1.3) we can consider an event of light being produced in an instant t = t' = 0 at point P of the following figure :



(Fig. A.3)

Questions :

1) What are the coordinates of this event for each of the reference points ?
2) If one accepts the slowing of the mobile clock, what ratios link the two scale marks ?

Answers :

The coordinates :

For R :

$$x = ct$$
$$x_1 = vt$$
$$x_2 = x - x_1 = (c - v)t$$

For R' :

$$x_1' = vt'$$
$$x_2' = ct'$$
$$x' = x_1' + x_2' = (c + v)t'$$

The ratios :

LORENTZ Ratios :

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{(A.5a)}$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{(A.5b)}$$

Establishing the ratios :

R' translates the slowing of the clock by : $t' \leftarrow \dfrac{t'}{\sqrt{1 - \frac{v^2}{c^2}}}$.

This produces :

$$x_1 = \frac{vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_2 = \frac{ct'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x = x_1 + x_2 = \frac{(c+v)t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$\Rightarrow \quad t' = x \dfrac{\sqrt{1 - \frac{v^2}{c^2}}}{c+v}$; of the replacement $x \leftarrow ct$ the following is deduced :

$$t' = t \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \qquad \text{(A.6a)}$$

$$\Rightarrow \qquad x_2' = x' = ct \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$$

$$x' = x \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \qquad \text{(A.6b)}$$

It is obvious that the substitution $x \leftarrow ct$, twice in (A.5a) and once in (A.5b), refers immediately to the (A.6) ratios. This shows unequivocally that, in uniform translation, the coordinates transform independently of each other.

A.5 MAXWELL Equations

Formalism :

MAXWELL equations[1] consist of the mathematical context of electrical and magnetic phenomena. On the left, they consist of four vectors of fields and on the right of four sources, only two of which are vectorial. They are written :

$$\vec{\nabla} \wedge \vec{E} + \frac{\partial \vec{B}}{\partial t} = -\vec{M} \tag{A.7a}$$

$$\vec{\nabla} \wedge \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J} \tag{A.7b}$$

$$\vec{\nabla} \cdot \vec{D} = \rho \tag{A.7c}$$

$$\vec{\nabla} \cdot \vec{B} = \tau \tag{A.7d}$$

with : $\vec{E}$ intensity of the electrical field ;
$\vec{D}$ electrical displacement ;
$\vec{B}$ magnetic induction ;
$\vec{H}$ intensity of the magnetic field or field auxiliary magnetic field ;
$\vec{J}$ surface density[2] of the electric current ;
$\vec{M}$ surface density of the magnetic current ;
ρ volumic density of the electrical charge ;
τ volumic density of the magnetic charge.

It should also be realised that :

- the system (A.7) is local and all the terms are functions of (s; t) ;

(1) established in 1872, published in 1873.
(2) This density (whose current pass over a surface or section) is not to be confused with the surface sources produced in the discontinuity zones.

- $\vec{\nabla} \wedge \vec{H} = \vec{J}$ is AMPERE's law (1820) ;
- $\vec{\nabla} \wedge \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$ is FARADAY's law (1831) ;
- the vector $\vec{D}$ introduced by MAXWELL in 1862 under the name of "electrical induction", $\frac{\partial \vec{D}}{\partial t}$ is known as a "displacement current" ;
- the terms $\vec{M}$ and τ are fictitious sources incorporated in the equations to simplify their resolution ;
- ρ exclusively represents free charges, the (fictitious) polarisation charges being included in $\vec{D}$ and $\vec{B}$;
- $\vec{J}$ it is the current for free (conduction and convection) charges.

Continuity equations :

It can easily be deduced from the (A.7), system that :

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \qquad \text{(A.8a)}$$

$$\vec{\nabla} \cdot \vec{M} + \frac{\partial \tau}{\partial t} = 0 \qquad \text{(A.8b)}$$

Structural equations :

These are equations that link $\vec{E}$ to $\vec{D}$ and $\vec{H}$ to $\vec{B}$ in order to render the (A.7) system soluble. They indicate the electrical and magnetic properties of the medium without the sources. They are experimental and generally integro-differential.

For an "instantaneous" and "local" medium, these equations are expressed as :

$$\vec{D} = \varepsilon \vec{E} \qquad \text{(A.9a)}$$

$$\vec{B} = \mu \vec{H} \qquad \text{(A.9b)}$$

$\varepsilon = \varepsilon(\vec{E}; s; t)$ and $\mu = \mu(\vec{H}; s; t)$ designate respectively the dielectric permittivity and the magnetic permeability of the medium.

In particular :

- if ε and μ are independent of the field, one can speak of a linear medium; as a result :

$$\varepsilon = \varepsilon(s; t) \text{ and } \mu = \mu(s; t) ;$$

- if additionally the medium is isotropic and homogenous, this becomes :

$$\varepsilon = \varepsilon(t) \text{ and } \mu = \mu(t) ;$$

- furthermore, if ε and μ are independent of time, the medium is known as "simple", and produces :

$$\varepsilon = \varepsilon_r\varepsilon_o \text{ and } \mu = \mu_r\mu_o$$

where ε_r and μ_r are specific, bitmapped constants that are generally complex, they are the equivalent of a unit in a vacuum.

Comment :

1) The linearity of the medium implies that of the system (A.7). This means that the effects of the sources (the fields) are rendered superposable.
2) The presence of known sources in an homogenous medium does not affect its homogeneity.

Electromagnetic power :

The "surface density of electromagnetic power" or the "intensity of the flow of energy" is known as POYNTING's vector (1884) :

$$\vec{P} = \vec{E} \wedge \vec{H} \text{ [W/m}^2\text{]} = \frac{1}{2}(\varepsilon E^2 + \mu H^2)\ \vec{v} ;$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} \text{ (propagation speed).}$$

In particular, for $v = c$, the result is : $\vec{E} \cdot \vec{H} = 0$

$$\Rightarrow \varepsilon_o E^2 = \mu_o H^2$$

Comment :

This power of diffusion is manifestly different from that of penetration according to PLANCK (I.6a). The first is a function of several variables, such as the power of the source, whereas the second only depends on the frequency.

A.6 FOURIER Transforms

Specific notes on the subject :

Abbreviations :

TF Direct FOURIER Transforms
TF^{-1} Inverse FOURIER Transforms

Notations :

$\vec{f}$ radius of FOURIER space vector :

$$\vec{f} = f_i \, \vec{u}_i \;\; ; \;\; (i = 1, 2, \ldots, N)$$

$\xrightarrow{\mathcal{F}}$ application of TF

$\xrightarrow{\mathcal{F}^{-1}}$ application of TF^{-1}

Written convention :

Unless explicitly stated otherwise, FOURIER Transforms of functions are printed in bold.

A.6.1 Analogical Transforms

Definitions :

If x is a real variable and g(x) a periodic function (or distribution) satisfying the DIRICHLET criteria or if it is aperiodic and locally integrable[1], the TF of such a function is expressed as :

$$g(x) \xrightarrow{\mathcal{F}} \mathbf{g}(f) \triangleq \int_{-\infty}^{\infty} g(x) \exp(-j2\pi fx)\, dx$$

$$\mathbf{g}(f) \xrightarrow{\mathcal{F}^{-1}} g(x) \triangleq \int_{-\infty}^{\infty} \mathbf{g}(f) \exp(j2\pi fx)\, df$$

(1) in the meaning of RIEMANN for the functions and LEBESGUE-integrable for the distributions.

Where $g(\vec{x})$ is a function at N independent variables, it becomes :

$$g(\vec{x}) \xrightarrow{\mathcal{F}} \mathbf{g}(\vec{f}) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} g(\vec{x}) \exp(-j2\pi \vec{f} \cdot \vec{x})\, dx_1 \dots dx_N$$

$$\mathbf{g}(\vec{f}) \xrightarrow{\mathcal{F}^{-1}} g(\vec{x}) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \mathbf{g}(\vec{f}) \exp(j2\pi \vec{f} \cdot \vec{x})\, df_1 \dots df_N$$

Bijection property :

If $g_1(x) = g_2(x)$ then $\mathbf{g}_1(f) = \mathbf{g}_2(f)$ and the inverse.

Domain of validity :

Apart from mathematics, FOURIER Transforms are valid for any ratio in physics.

Applications :

DIRAC functions :

$$\delta(x - pX) \xrightarrow{\mathcal{F}} \int_{-\infty}^{\infty} \delta(x - pX) \exp(-j2\pi fx)\, dx = \sum_{p=-\infty}^{\infty} \exp(-j2\pi f \times pX)$$

$$= \frac{1}{X}\, \delta\left(f - \frac{k}{X}\right) \; ; \; (k = 0, \pm 1, \pm 2, \dots, \pm\infty) \qquad \text{(A.10a)}$$

In particular, for p = 0, the result is :

$$\delta(x) \xrightarrow{\mathcal{F}} 1 \xrightarrow{\mathcal{F}^{-1}} \int_{-\infty}^{\infty} \exp(j2\pi fx)\, df = \delta(x) \qquad \text{(A.10b)}$$

This being the case, the identities are designated :

$$\delta(x) \equiv \int_{-\infty}^{\infty} \exp(j2\pi fx)\, df \qquad \text{(A.11a)}$$

$$\delta(f) \equiv \int_{-\infty}^{\infty} \exp(-j2\pi fx)\, dx \qquad \text{(A.11b)}$$

Convolutions :

$$g_1(\vec{x}) * g_2(\vec{x}) \xrightarrow{\mathcal{F}} \mathbf{g}_1(\vec{f}) \times \mathbf{g}_2(\vec{f}) \qquad \text{(A.12a)}$$

$$\mathbf{g}_1(\vec{f}) * \mathbf{g}_2(\vec{f}) \xrightarrow{\mathcal{F}^{-1}} g_1(\vec{x}) \times g_2(\vec{x}) \qquad \text{(A.12b)}$$

Periodic functions :

$$g\left(x_1 - p_1X_1, ..., x_N - p_NX_N\right) \ ; \ (p_1, p_2, \ldots, p_N = 0, \pm 1, \pm 2, \ldots, \pm\infty)$$

$$\xrightarrow{\mathcal{F}} \mathbf{g}(\vec{f}) \exp\left(-j2\pi \vec{f} \cdot \vec{X}\right) \qquad \text{(A.13)}$$

Derivatives :

Mono-variables functions :

$$\frac{dg(x)}{dx} \xrightarrow{\mathcal{F}} j2\pi f \times \mathbf{g}(f) \qquad \text{(A.14a)}$$

Demonstration :

$$\mathbf{g}(f) = \int_{-\infty}^{\infty} g(x) \exp(-j2\pi fx)\, dx \ \Rightarrow \ \frac{d\mathbf{g}(f)}{dx} = 0$$

$$\frac{d\mathbf{g}(f)}{dx} = \int_{-\infty}^{\infty} \left[\frac{dg(x)}{dx} \exp(-j2\pi fx) - j2\pi f\, g(x) \exp(-j2\pi fx)\right] dx$$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{dg(x)}{dx} \exp(-j2\pi fx)\, dx = j2\pi f \int_{-\infty}^{\infty} g(x) \exp(-j2\pi fx)\, dx$$

Hence the ratio in view.

Generalisation : $$\frac{d^n g(x)}{dx^n} \xrightarrow{\mathcal{F}} (j2\pi f)^n\, \mathbf{g}(f) \qquad \text{(A.14b)}$$

Demonstration of the formula (III.6a) :

The application of TF to the two members of the formula implies :

$$(j2\pi f)^n\, \mathbf{g}(f) = \mathbf{g}(f) \times (j2\pi f)^n$$

This equality and the bijection property prove the formula in question.

Multivariable functions :

$$\frac{\partial^m g(\vec{x})}{\partial^{m_1} x_1 \ldots \partial^{m_N} x_N} \quad ; \quad (m = m_1 + m_2 + \ldots + m_N)$$

$$\xrightarrow{\mathcal{F}} (j2\pi)^m f_1^{m_1} \ldots f_N^{m_N} \mathbf{g}(\vec{f})$$

Vectorial derivatives :

$$\vec{g}(\vec{x}) = \sum_{i=1}^{N} g_i(\vec{x})\vec{u}_i \xrightarrow{\mathcal{F}} \vec{\mathbf{g}}(\vec{f}) = \sum_{i=1}^{N} \mathbf{g}_i(\vec{f})\vec{u}_i$$

$$\vec{\nabla} \times g(\vec{x}) \xrightarrow{\mathcal{F}} j2\pi\vec{f} \times \mathbf{g}(\vec{f})$$

$$\vec{\nabla} \cdot \vec{g}(\vec{x}) \xrightarrow{\mathcal{F}} j2\pi\vec{f} \cdot \vec{\mathbf{g}}(\vec{f})$$

$$\vec{\nabla} \wedge \vec{g}(\vec{x}) \xrightarrow{\mathcal{F}} j2\pi\vec{f} \wedge \vec{\mathbf{g}}(\vec{f})$$

A.6.2 Digital Transforms

The TF of a digital function is analogic :

$$g(m\Delta x) \xrightarrow{\mathcal{F}} \mathbf{g}(f)$$

The digitisation of the latter is taken from the substitution $f \leftarrow w\Delta f$:

$$\mathbf{g}(w\Delta f) \triangleq \mathbf{g}(f)\,\delta(f - w\Delta f) \; ; \; (w = 0, \pm 1, \pm 2, \ldots, \pm\infty) \qquad \text{(A.15)}$$

Property of periodicity :

The TF of digital functions are periodic : TF in $\frac{1}{\Delta x}$ and TF^{-1} in $\frac{1}{\Delta f}$.

Corollary :

The periodicity of $\mathbf{g}(f)$ in $\frac{1}{\Delta x}$ terminates the variation at :

$$W = \frac{1}{\Delta x \times \Delta f} \tag{A.16}$$

The DIRAC set :

$$\delta(x - p\nu\Delta x) \xrightarrow{\mathcal{F}} \int_{-\infty}^{\infty} \delta(x - p\nu\Delta x)\exp(-j2\pi fx)\,dx$$

$$= \Delta x \sum_{p=-\infty}^{\infty} \exp(-j2\pi f \times p\nu\Delta x) = \frac{1}{\nu}\,\delta\Big(f - \frac{k}{\nu\Delta x}\Big)\;;$$

$$(k = 0, \pm1, \pm2, \ldots, \pm\infty)$$

Typical functions :

Mono-variable function :

$$g(m\Delta x) \xrightarrow{\mathcal{F}} \mathbf{g}(f) = \int_{-\infty}^{\infty} g(x)\,\delta(x - m\Delta x)\exp(-j2\pi fx)\,dx$$

$$= \Delta x \sum_{m=-\infty}^{\infty} g(m\Delta x)\exp(-j2\pi f \times m\Delta x)$$

The substitution $f \leftarrow w\Delta f$, the (A.16) ratio and the $m \in [0, M]$ interval induce the final expression of the digital TF :

$$\mathbf{g}(w\Delta f) = \Delta x \sum_{m=0}^{M} g(m\Delta x)\exp\Big(-j2\pi\frac{wm}{W}\Big)\;;$$

$$(w = 0, 1, 2, \ldots, W) \tag{A.17a}$$

As for TF^{-1}, the same procedure involves :

$$\mathbf{g}(w\Delta f) \xrightarrow{\mathcal{F}^{-1}} g(x) = \int_{0}^{\frac{1}{\Delta x}} \mathbf{g}(f)\,\delta(f - w\Delta f)\exp(j2\pi fx)\,df$$

$$= \Delta f \sum_{w=0}^{W} \mathbf{g}(w\Delta f)\exp(j2\pi xw\Delta f)$$

This, the substitution $x \leftarrow m\Delta x$ and the equality (A.16) give :

$$g(m\Delta x) = \frac{1}{W\Delta x} \sum_{w=0}^{W} \mathbf{g}(w\Delta f) \exp\left(j2\pi \frac{wm}{W}\right) ;$$

$$(m = 0, 1, 2, \ldots, M) \qquad \text{(A.17b)}$$

On the other hand, the periodicity of $g(m\Delta x)$ $\frac{1}{\Delta f}$ releases the equality thereof :

$$M\Delta x = \frac{1}{\Delta f} \qquad \text{(A.18)}$$

The latter and (A.16) equalise M to W. In this, the number of measures in the original space is always the same as in the FOURIER space.

Multivariable function :

The generalisation of the transforms (A.17) into N variables is imminent :

$$\mathbf{g}(w_1\Delta f_1, \ldots, w_N\Delta f_N) = \Delta x_1 \ldots \Delta x_N \sum_{m_1=0}^{M_1} \ldots$$

$$\sum_{m_N=0}^{M_N} g(m_1\Delta x_1, \ldots, m_N\Delta x_N) \exp\left[-j2\pi\left(\frac{w_1 m_1}{W_1} + \ldots + \frac{w_N m_N}{W_N}\right)\right] ;$$

$$(w_1 = 0, 1, 2, \ldots, W_1; \ \ldots ; \ w_N = 0, 1, 2, \ldots, W_N) \qquad \text{(A.19a)}$$

$$g(m_1\Delta x_1, \ldots, m_N\Delta x_N) = \frac{1}{W_1\Delta x_1 \ldots W_N \Delta x_N} \sum_{w_1=0}^{W_1} \ldots$$

$$\sum_{w_N=0}^{W_N} \mathbf{g}(w_1\Delta f_1, \ldots, w_N\Delta f_N) \exp\left[j2\pi\left(\frac{w_1 m_1}{W_1} + \ldots + \frac{w_N m_N}{W_N}\right)\right] ;$$

$$(m_1 = 0, 1, 2, \ldots, M_1; \ \ldots ; \ m_N = 0, 1, 2, \ldots, M_N) \qquad \text{(A.19b)}$$

Convolution :

The result of the ratios (III.3b) and (A.12) is :

$g_1(m_1\Delta x_1, \ldots, m_N\Delta x_N) * g_2(m_1\Delta x_1, \ldots, m_N\Delta x_N)$

$\xrightarrow{\mathcal{F}}$ $\mathbf{g}_1(w_1\Delta f_1, \ldots, w_N\Delta f_N) \times \mathbf{g}_2(w_1\Delta f_1, \ldots, w_N\Delta f_N)$

The TF^{-1} can be found in the same way.

Derivatives :

In the same way that the (A.14) formulae were demonstrated, the following is deduced :

$$\delta^{(n)}(x - m\Delta x) \xrightarrow{\mathcal{F}} (j2\pi f)^n \, \delta\left(f - \frac{w}{\Delta x}\right) ;$$

$$(w = 0, \pm 1, \pm 2, \ldots, \pm\infty) \qquad \text{(A.20a)}$$

$$g^{(n)}(m\Delta x) \xrightarrow{\mathcal{F}} (j2\pi f)^n \, \mathbf{g}(f) \, \delta(f - w\Delta f) \qquad \text{(A.20b)}$$

Demonstration of the (III.6b) ratio :

The application of TF to the two members of the said ratio produces equality :

$$(j2\pi f)^n \, \mathbf{g}(f) = \mathbf{g}(f) \, (j2\pi f)^n$$

This and the bijection property corroborate the formula (III.6b).

Digital values

Constants :

Fundamental constants :

$$\varepsilon_0 = \frac{10^{-9}}{36\pi} \; [\text{js}^4/\text{m}^5 = \text{F/m}]$$

$$\mu_0 \; \underline{\Delta} \; 4\pi \times 10^{-7} \; [\text{m}^3/\text{js}^2 = \text{H/m}]$$

$$\gamma_0 \simeq 1.19261 \times 10^{9} \; [\text{js}^4/\text{m}^5] \; \Rightarrow$$

$$G = \frac{1}{4\pi\gamma_o} \simeq 6.67256 \times 10^{-11} \; [\text{m}^5/\text{js}^4]$$

Fundamental measurements :

$$\mathbb{h} \simeq 6.626176 \times 10^{-34} \; [\text{j}]$$

$$H_0 \simeq 2.427185 \times 10^{-18} \; [1/\text{s}]$$

Absolute constants :

$$a \simeq 2.4386333 \times 10^{-2} \; [\text{m/j}]$$

$$b \simeq 1.22934619666 \times 10^{10} \; [\text{j/s}]$$

$$c \; \underline{\Delta} \; 3 \times 10^{8} \; [\text{m/s}]$$

Measurements :

$$h \simeq 6.626176 \times 10^{-34} \; [\text{js}] \; \Rightarrow \; \hbar = \frac{h}{2\pi} \simeq 1.054589 \times 10^{-34} \; [\text{js}]$$

$$k \simeq 1.380662 \times 10^{-23} \; [\text{js}]$$

$$\mathbb{k} \simeq 1.380662 \times 10^{-23} \; [\text{j}]$$

Limits :

PLANCK's natural units :	*HUBBLE's cosmic units :*
$\Delta t \simeq 5.39 \times 10^{-44}$ [s]	$\Delta T \simeq 4.12 \times 10^{17}$ [s]
$\Delta s \simeq 1.616 \times 10^{-35}$ [m]	$\Delta S \simeq 1.236 \times 10^{26}$ [m]
$\Delta e \simeq 6.626176 \times 10^{-34}$ [j]	$\Delta E \simeq 5.0649 \times 10^{27}$ [j]

Large circles of the universe :

$\mathbf{T} \simeq 4\pi \times 10^{78}$ [s]

$\mathbf{S} \simeq 12\pi \times 10^{86}$ [m] ➔

hypothetical radius of the universe : $\mathbf{R} \simeq 6 \times 10^{86}$ [m]

$\mathbf{E} \simeq 1.54482 \times 10^{89}$ [j]